Exchange Rates and Asset Prices in An Open Economy with Rare Disasters*

Kai Guo†
Department of Economics
Harvard University

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Abstract

By introducing rare but severe disasters into an otherwise standard open-economy general equilibrium model and allowing the disaster probability to be both time-varying and mean-reverting, several macroeconomics, finance and international finance puzzles can be explained in a single model. The puzzles include the equity premium puzzle, the risk-free rate puzzle, the forward discount puzzle, the excess volatility puzzle and the volatility mismatch puzzle. A mean-reverting disaster probability also generates return predictability and the leverage effect in the stock market. The model, when calibrated with plausible parameter values, can replicate many salient features in the stock price and exchange rate data. The model maintains good tractability by having a representative agent, time-additive and isoelastic preferences and complete markets. Closed-form solutions can be obtained under certain conditions. Finally, the asset pricing implications of rare disasters under the Epstein-Zin-Weil preferences are studied. Besides explaining several above puzzles, a novel implication is that higher returns in the Home stock market relative to the Foreign stock market are associated with a Home currency depreciation, a stock market version of the uncovered interest parity condition which is consistent with empirical findings.

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†Tel: 617-495-1129. Email: kaiguo@fas.harvard.edu

Only Thing We Have to Fear Is Fear Itself. – Franklin D. Roosevelt

The Further Back I Look, The Further Forward I Can See. – Winston Churchill

1 Introduction

Many puzzles in macroeconomics, finance and international finance look alike. Both stock prices and exchange rates seem to be too volatile to be explained by fundamentals. Both the equity premium and the risk premium in the foreign exchange market are time-varying and seem to be too high to be justified by associated risks. High stock prices predict low future returns and high interest rate currencies on average appreciate. But sometimes these puzzles also contradict each other. For instance, exchange rates may be too volatile when looking at consumption data but they become too smooth when compared to stock prices. Given all these similar yet sometimes conflicting puzzles, it may be useful to think about all of them in a single, coherent model. This paper achieves this with some success.

In this paper, as many standard features as possible are preserved: a representative agent, isoelastic and time-addititive preferences and complete markets. These features keep the model tractable and also make the mechanisms transparent. Yet, the key departure of this paper from the standard RBC or asset pricing model is the existence of rare disasters following Rietz (1988) and more recently Barro (2006). It can be viewed as an open-economy extension of Barro (2006) with several important distinctions. The first distinction is naturally the open-economy setup, which allows me to examine simultaneously issues about the stock market and exchange rates. Technically speaking, extending a model from a closed-economy setting to an open-economy one is not always simple. A strength of the paper is that analytical tractability is kept as much as possible. Many results can be easily derived with paper and pencil. The second deviation from Barro (2006) is that instead of having a fixed disaster probability, this paper makes the disaster probability time-varying. This is meant to capture the idea that people's perception about future risks do vary over time. Time-varying disaster probabilities generate interesting dynamics and help explain several puzzles that are not addressed in Barro (2006). In order to study the forward discount puzzle, which is traditionally a puzzle concerning nominal variables, this paper also explicitly introduces money into the model. Two different monetary regimes are considered–a monetarist world and a fiscalist world. It is reassuring that the results of the model do not depend on how the nominal side is modeled. However, some quantitative implications do depend on what kind of monetary policy (or fiscal policy in the fiscalist world) is adopted.

The following features can be generated from the model: a low risk-free rate and a high equity premium; low stock prices predicting high and more volatile future returns; volatile and persistent stock prices; volatile exchange rates but less so than stock prices; and low interest rate currencies depreciating on average. Thus, this paper provides a coherent explanation of the equity premium puzzle, the risk-free rate puzzle, the excess volatility puzzle, the forward discount puzzle and the volatility mismatch puzzle both qualitatively and quantitatively.

How does this model work exactly? Simply speaking, rare disasters make stocks much riskier to hold as dividends drop sharply during a disaster when consumption is also very low and thus create a high equity premium. On the other hand, precautionary saving motive due to the existence of rare disasters increases demand for safe assets and hence lowers the risk-free rate. So rare disasters can account for both high equity premia and low risk-free rates at the same time. In the model, stock prices are functions of disaster probability. Persistent and mean-reverting disaster probability generates persistent and mean-reverting stock prices, which in turn leads to long term predictability of stock returns–high stock price today indicates lower stock price tomorrow therefore high stock price predicts lower future returns. Time-varying disaster probability also causes volatile stock prices because small but persistent changes in the disaster probability can substantially increase/decrease the cumulative probability of being hit by disasters in the long run and therefore causes "excess volatility" in stock prices. With respect to exchange rates, rare disasters generate the "safe-haven" effect in the foreign exchange market–currencies that are considered safer appreciate and pay lower interest rates. But as long as such safe-haven effect is not permanent, low interest currencies depreciate on average. This is exactly what the forward discount puzzle is about. This model generates volatile exchange rates by having a reasonably concave utility function and a low correlation of consumption growth across countries. Real exchange rates are approximately equal to consumption differentials multiplied by the coefficient of relative risk aversion. A low cross-country consumption correlation ensures that there is considerable volatility in consumption growth differentials and a more concave utility function naturally results in more volatile exchange rates, *ceteris paribus*. As for the volatility mismatch puzzle, the volatility of stock prices is mainly driven by time-varying disaster probability while the volatility of exchange rates is mostly due to relative consumption growth between Home and Foreign. Namely, we can have two sets of parameters to match the volatility of stock prices and the volatility of exchange rates separately. So there is no volatility mismatch problem.

It is important to point out that the way this model works is not the so-called "peso problem". Even when actual disasters are allowed in the simulation, none of the quantitative results change significantly. It is not disasters that matter, but rather the fear of disasters that drives everything.

Finally, the Epstein-Zin-Weil preferences are introduced. This allows the coefficient of relative risk aversion and the intertemporal elasticity of substitution (IES) to be separately parameterized and therefore it can get rid of a counterintuitive and probably counterfactual asset pricing implication of the time-separable utility–a higher disaster probability causes stock prices to rise not to fall. With the Epstein-Zin-Weil preferences, the model can still explain the equity premium puzzle, the risk-free rate puzzle and the forward discount puzzle. It can also generate return predictability and the leverage effect in the stock market. However, it does not match the volatility of stock prices and exchange rates very well, possibly a result of some oversimplifying assumptions. In addition, a novel implication of the Epstein-Zin-Weil preferences is that higher returns in the Home stock market relative to the Foreign stock market are associated with a Home currency depreciation when both the coefficient of relative risk-aversion and the IES are greater than 1. This prediction is consistent with empirical findings in Hau and Rey (2006). Therefore, with the Epstein-Zin-Weil preferences and rare disasters, this model can allow the uncovered interest parity condition to fail in the bond market but to hold in the stock market.

The remaining of this paper is organized as follows: Section 2 provides a brief review of the relevant puzzles and existing explanations. Section 3 introduces the model and solutions. Section 4 calibrates and quantitatively evaluates the model. Section 5 discusses the model intuition in more detail. Section 6 conducts various robustness checks. Section 7 introduces the Epstein-Zin-Weil preferences and section 8 concludes. Appendix contains detailed derivations.

2 Review of Puzzles and Related Literature

Since most puzzles in this paper have already been well-established in the literature, and given the enormity of the literature from both an empirical and theoretical perspective, I confine my attention only to the papers that also attempt to provide a unifying explanation to the above puzzles. Since this paper maintains the rational expectation assumption, papers that have relaxed the assumption of (constrained) rational expectation are also not included.

The equity premium puzzle was first pointed out by Mehra and Prescott (1985), that mean excess returns to stocks were too high to be rationalized with the observed risks associated with stocks unless people were extremely risk-averse. If one is willing to accept the notion that people are indeed extremely risk-averse, the flip-side of the equity premium puzzle is the risk-free rate puzzle, first pointed out by Weil (1989). For an agent with the standard isoelastic time-separable utility, high risk aversion implies a low intertemporal elasticity of substitution (IES) and in a growing economy this would lead to an incredibly high real interest rate, which is not true in the data. For this reason, the equity premium puzzle and the risk-free rate puzzle are a pair of puzzles that must be addressed together.

Excess volatility: Stock prices are very volatile and to a lesser extent exchange rates are also quite volatile. The excess volatility of stock prices was first documented by Shiller(1981) and LeRoy and Porter(1981) in a standard present-value framework. The volatility of exchange rates became a question of interest even earlier. Dornbusch had his "overshooting" paper published in 1976, one of the main themes of that paper is to explain why exchange rates are so volatile under the flexible exchange rate regime. Whether volatility is excess or not is certainly a subject of debate, but nevertheless, a model that matches the empirical volatilities of both stock prices and exchange rates is certainly more desirable.

The forward discount puzzle (a.k.a. the forward premium puzzle or the UIP puzzle) violates an important pillar in the international finance literature–the uncovered interest parity (UIP) condition. The UIP condition states that the interest differentials between two currencies should be able to predict future exchange rate movements. A currency with a higher interest rate should be expected to depreciate–otherwise there would be arbitrage opportunities. Despite the miserable predicting power of interest rate differentials on exchange rates, the UIP condition does not even get the sign correct.[1] Scores of research have repeatedly shown that among major currencies a currency with a higher interest rate on average *appreciates*, not depreciates. More specifically, the UIP condition predicts that running the regression below should yield a point estimate of $\beta_1 = 1$. However an overwhelming majority of

[1] See Meese and Rogoff(1983) and more recently Cheung el al. (2002).

studies find that $\beta_1 < 0$, albeit not always significantly different from 0.[2]

$$e_{t+1} - e_t = \beta_0 + \beta_1(i_{t+1} - i^*_{t+1}) + \epsilon_t \tag{1}$$

The regression above is also called the Fama regression, where e_t is the logarithm of nominal exchange rate defined as the number of units of Home currency per unit of Foreign currency and i_{t+1} and i^*_{t+1} are Home and Foreign nominal interest rates between period t and $t+1$. It is noteworthy that the forward discount puzzle is not only a statistical puzzle but also an investment strategy known as the carry trade. Burnside, Eichenbaum, Kleshchelski and Rebelo (2006) and Lustig and Verdelhan (2007a) both show that returns to carry trade are quite high, although disagreeing on whether such returns are compensations for risks or not.[3]

The volatility mismatch puzzle has been recently pointed out by Brandt, Cochrane and Santa-Clara (2006). The puzzle is stated as following: using the Hansen and Jagannathan (1991) bound to quantify the volatility of marginal utility growth (or stochastic discount factor) from stock price data would give a number of around 50%. But using exchange rate data to back out the volatility of relative marginal utility growth yields something around 15%, i.e. we have the following two conditions:

$$std(\ln m_{t+1}) \simeq std(\ln m^*_{t+1}) \simeq 50\%$$

and

$$std(\ln m_{t+1} - \ln m^*_{t+1}) \simeq 15\%$$

where m_{t+1} and m^*_{t+1} are the Home and Foreign stochastic discount factors. If both conditions were to be satisfied, the marginal utility growth would have to be highly correlated across countries. This in turn, as the logic goes, would imply that international risk-sharing is nearly perfect. However, such high degree of risk-sharing can hardly be seen in the consumption data. Furthermore, they argue that introducing trade cost or incomplete market would not change the results. Since this puzzle is not a specific feature of any particular model, I view it as a major challenge to any C-CAPM model

[2] Froot and Thaler (1990) point out that for 75 published estimates of equation 1, the average point estimate of β_1 is -0.88. More recent researches have generally obtained similar findings, see Backus et al (2001).

[3] See Burnside(2007) for a critique of Lustig and Verdelhan (2007a) and their response in Lustig and Verdelhan(2007b).

that tries to understand stock prices and exchange rates simultaneously. Any successful C-CAPM model should be able to generate very volatile stock prices, quite volatile exchange rates and a low correlation in the consumption growth.

There are many theories proposed in the literature that try to address the aforementioned puzzles. Among theories that maintain the representative agent, complete market and perfect information assumptions, two lines of researches stand out as potential unifying explanations for all of the puzzles above.

One line of research is the "habit-formation" approach starting from Constantinides (1990). Campbell and Cochrane (1999) is a version of this approach and is able to match the U.S. stock price data quite well. Verdelhan (2007) extends Campbell and Cochrane (1999) to an open economy model and provides a potential explanation of the forward discount puzzle.

The other line of research is the "risks for the long run" approach first proposed by Bansal and Yaron(2004). Bansal and Yaron (2004) are able to explain the equity premium puzzle and the risk-free rate puzzle. Colacito and Croce (2005) extend this approach to an open economy setting to address the volatility mismatch puzzle and Bansal and Shaliastovich (2007) further extend Colacito and Croce (2005) to explain the forward discount puzzle.

There is also a large literature that relaxes the standard assumptions such as representative agent, complete information or complete market. An incomplete list includes segmented market, limited participation, limited enforceability, heterogeneous agent, idiosyncratic and uninsurable income risk, borrowing constraints and limited information. See Mehra and Prescott (2003) and references therein for relevant literature on the equity premium puzzle and Alvarez, Atkeson and Kehoe (2007) and Bacchetta and Van Wincoop (2006) for potential explanations of the forward discount puzzle along this line of thought.

Most closely related to this paper are Gabaix (2007) and Farhi and Gabaix (2007). They have a set of models that explain several macro/finance puzzles by introducing rare disasters. Martin (2007) also allows for rare disasters.

3 A Stochastic Open Economy Model with Time-Varying Disaster Probability

This model is an open economy version of Lucas (1978) and Mehra and Prescott (1985). It allows for rare but severe disasters in the same way as Rietz (1988) and more recently Barro (2006). Besides its open economy nature, another important departure from Barro (2006) is that the probability of rare disasters is assumed to be a persistent time-varying process. This is to capture the notion that people's perception about risks of potential disasters do seem to vary over time. Even Mehra and Prescott (1988), who considered Rietz (1988) as an implausible explanation for the equity premium puzzle, admitted that the perceived probability of extreme events should probably be time-varying.[4] It must be emphasized that the objective of this paper is not to track the exact evolution of disaster probabilities over time and thereby match the observed time series of asset prices. In other words, this paper won't answer questions such as why interest rates were so low at a particular point in time or why the U.S. dollar depreciated at another point in time. Rather, this paper is about trying to illustrate how a time-varying disaster probability may provide a useful way to think about several domestic and international asset-pricing puzzles in one model, by showing that how the simulated moments from the model can match important moments in the exchange rate and stock price data quite well.

3.1 The World Economy, Preferences and Endowments

The world economy consists of a continuum of small open economies.[5] All countries are completely symmetric unless otherwise mentioned. Pick two arbitrary countries, call them Home and Foreign. A Home representative agent has the following preferences

[4] Mehra and Prescott (1988, p. 135) wrote: "...the perceived probability of a recurrence of a depression was probably high just after World War II and then declined..." and "...Similarly, if the low-probability event precipitating the large decline in consumption were a nuclear war, the perceived probability of such an event surely has varied in the last 100 years..."

[5] The small open economy assumption is made mostly for technical considerations. It significantly simplifies the algebra and makes closed-form solutions possible. Also, it reduces the number of state variables from three (relative size of countries, Home disaster probability and Foreign disaster probability) to only one (Home disaster probability) and therefore avoid the curse of dimensionality. An earlier version of this paper considers a two-country case, in which all results presented below carry through.

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\epsilon} \left(\frac{M_s}{P_s} \right)^{1-\epsilon} \right] \right\} \tag{2}$$

where C is an index of per capita consumption of a continuum of goods indexed by $i \in [0, 1]$,

$$C = \exp(\int_0^1 \log c_i di) \tag{3}$$

Among all consumption goods, a α fraction of the goods is non-tradable goods and the remaining fraction $1 - \alpha$ is tradable goods. Tradable goods are identical across all countries.[6] Ranking goods according to their tradability so that $0 \leq i \leq \alpha$ is for non-tradable goods and $\alpha < i \leq 1$ is for tradable goods, the consumption index can be rewritten as

$$C = \exp(\underbrace{\int_0^\alpha \log c_i di}_{Non-Tradable} + \underbrace{\int_\alpha^1 \log c_i di}_{Tradable}) \tag{4}$$

Later, when symmetry is imposed, the consumption index can be further simplified to the following Cobb-Douglas representation

$$C = c_N^\alpha c_T^{1-\alpha} \tag{5}$$

where $c_i = c_N$ for $\forall i \in [0, \alpha]$ and $c_i = c_T$ for $\forall i \in (\alpha, 1]$.

Money is introduced through the standard money-in-utility specification following Obstfeld and Rogoff (2002), among many others. $\frac{M}{P}$ is the real money balance. This is the monetarist version of the model. In the appendix, a fiscalist version where the nominal price level is determined by fiscal policy à la Cochrane (2005) is also considered. These two approaches yield the same asset pricing implications for the purpose of this paper. Therefore, the remaining of this paper will focus on the monetarist version and only discusses the effects of fiscal policy on price level, inflation and exchange rate when necessary. Following the tradition of international economics, all Foreign variables are denoted with asterisks. Foreign agents have identical preferences except that they must hold their own national currency M^*, which is deflated by the Foreign price index P^*.

[6] Introducing different tradable goods will complicate the analysis by adding the terms of trade effect into the model. However, the terms of trade effect may be an important element in understanding international risk sharing, exchange rates and other international finance problems. See Cole and Obstfeld (1991), Corsetti and Pesenti (2001) and Pavlova and Rigobon (2007).

The Home representative agent owns a continuum of fruit trees that correspond to the consumption goods. In each period, fruit tree i produces y_{it} units of good i. y_{it} is a random variable whose realization depends on the state of nature, which will be specified later. Fruits can either be consumed or traded, but are not storable. Only tradable goods can be physically transported across national borders and there is no trade cost.

3.2 Prices and Budget Constraints

The consumption-based Home-currency price index in given by

$$P = \exp(\int_0^1 \log p_i di) \tag{6}$$

where p_i is the price of good i. Again under symmetry, the above price index will have a familiar Cobb-Douglas form

$$P = p_N^{\alpha} p_T^{1-\alpha} \tag{7}$$

where $p_i = p_N$ for $\forall i \in [0, \alpha]$ and $p_i = p_T$ for $\forall i \in (\alpha, 1]$. The Foreign price index is similarly defined.

The law of one price (LOOP) holds for all tradable goods, so that $p_i = \mathcal{E} p_i^*$ for $\forall i \in (\alpha, 1]$, where $\mathcal{E}$ is the nominal exchange rate. The presence of non-tradable goods means that the LOOP won't be true at the aggregate level. However, it is straightforward to show that the real exchange rate is simply the relative price of non-tradable goods between Home and Foreign. Under symmetry, the real exchange rate Q satisfies

$$Q = \mathcal{E}^{\alpha} (\frac{p_N^*}{p_N})^{\alpha} \tag{8}$$

Following Chari, Kehoe and McGrattan (2002), the markets for state-contingent money claim are assumed to be complete. In particular, there exists a complete set of one-period nominal Arrow-Debreu securities denominated in the Home currency. Let s_t denote the state of nature in period t and $Q(s_{t+1}|s_t)$ denote the price of security $B(s_{t+1})$ that delivers one unit of Home currency when state s_{t+1} occurs for each unit of this security, the Home representative agent faces the following budget constraints at period t in state s_t

$$P_t C_t + M_t + \sum_{s_{t+1}} Q(s_{t+1}|s_t) B(s_{t+1}) \leq P_t Y_t + M_{t-1} + B(s_t) + T_t \tag{9}$$

$$c_{it} \leq y_{it} \quad \forall i \in [0, \alpha] \tag{10}$$

where C_t, M_t and T_t are consumption, nominal money balance and transfers of Home money, respectively. Y_t is the index for endowments defined as $Y_t = \exp(\int_0^1 \log y_{it} di)$. Equation (10) is the feasibility constraint that requires consumption of non-tradable goods to not exceed available endowments at any point in time in any given state.

The Foreign representative agent has similar budget constraints except that everything is in terms of the Foreign currency

$$\begin{aligned} & P_t^* C_t^* + M_t^* + \sum_{s_{t+1}} Q(s_{t+1}|s_t) B^*(s_{t+1}) / \mathcal{E}_t \\ \leq \ & P_t^* Y_t^* + M_{t-1}^* + B^*(s_t)/\mathcal{E}_t + T_t^* \end{aligned} \tag{11}$$

$$c_{it}^* \leq y_{it}^* \quad \forall i \in [0, \alpha] \tag{12}$$

The Home government returns all seigniorage via lump-sum cash transfer, so that

$$T_t = M_t - M_{t-1} \tag{13}$$

Initial money and asset holding M_0 and $B(s_0)$ are assumed to be given. Foreign has similar conditions.

3.3 Shocks to the Economy

Only country-specific aggregate shocks are considered. This is an innocuous assumption for the purpose of this paper but may not be true under other contexts. The law of large numbers and complete markets ensure that idiosyncratic shocks to any specific good won't have any aggregate effects. Allowing shocks hitting the tradable sector to be different from those that hit the non-tradable sector turns out to have no effect on the results since in the end only the shocks to the non-tradable sector matter. In this regard, there is no need to distinguish aggregate shocks from sector-specific or good-specific shocks. Therefore assume

$$y_{it} = A_t \text{ for } \forall i \tag{14}$$

where A_t is the Home aggregate productivity.

Following Barro (2006), in addition to $i.i.d.$ normal shocks, there are also rare but severe disasters. Productivity growth at Home is described by

$$A_{t+1} = A_t e^{g+u_{t+1}+v_{t+1}} \tag{15}$$

where g is the trend growth rate, $u_{t+1} \sim N(0, \sigma_u^2)$ is the normal productivity shock and v_{t+1} picks up rare but severe disasters, which cause the whole economy to contract by proportion b were it to happen. The distribution of v_{t+1} is given by

$$\begin{aligned} v_{t+1} &= \log(1-b) \text{ with probability } 1 - e^{-p_{t+1}} \\ v_{t+1} &= 0 \text{ with probability } e^{-p_{t+1}} \end{aligned}$$

Different from Barro (2006), the disaster probability p_{t+1} is not fixed. Instead, it is assumed to be a simple AR(1) process drifting around a benchmark disaster probability p_{bench}

$$p_{t+1} - p_{bench} = \eta(p_t - p_{bench}) + \epsilon_t \tag{16}$$

I'll set $p_{bench} = 0.017$, which matches the empirical disaster probability among 35 countries in the 20th century documented in Barro (2006). Individual maximization also requires p_{t+1} to be not too big–otherwise the individual utility becomes unbounded. Therefore, I restrict $p_{t+1} \in [0.001, 0.025]$. Foreign has similar shocks and the Foreign disaster probability evolves according to the following equation

$$p^*_{t+1} - p_{bench} = \eta(p^*_t - p_{bench}) + \epsilon^*_t \quad (17)$$

ϵ_t and ϵ^*_t are assumed to be independently distributed.

3.4 Market Clearing and the Equilibrium Consumption Path

The way to proceed is to first solve out the equilibrium consumption path both in Home and Foreign. Once we know the equilibrium consumption path, pricing kernels or stochastic discount factors are well-defined and assets can then be priced.[7] All detailed derivations are included in the appendix, and here I only list the core equations. Home consumption of non-tradable goods and tradable goods are respectively

$$c_{it} = y_{it} \equiv A_t \ \forall i \in [0, \alpha] \quad (18)$$

$$c_{it} = \frac{A_t^{-\gamma}}{\int\limits_{w\in\Omega} A_{\omega t}{}^{-\gamma} d\omega} \int\limits_{w\in\Omega} A_{\omega t} d\omega \ \forall i \in (\alpha, 1] \quad (19)$$

where ω is the country index, Ω is the universe of all countries and $\gamma = \frac{(1-\rho)\alpha}{(\rho-1)\alpha-\rho} > 0$ when $\rho > 1$. The relative price between non-tradable goods and tradable goods is

$$\frac{p_N}{p_T} = \frac{\alpha}{1-\alpha} A_t^{-1-\gamma} \quad (20)$$

Equation (18) is simply that the Home agent must consume her own endowments of non-tradable goods. Equation (19) reflects the fact that the marginal utility of tradable goods consumption must be proportional across all countries at any time t and in any state s_t as a consequence of complete asset markets.[8] As a result, Home consumption of tradable goods is a fraction $\frac{A_t^{-\gamma}}{\int\limits_{w\in\Omega} A_{\omega t}{}^{-\gamma} d\omega}$ of the world total output $\int\limits_{w\in\Omega} A_{\omega t} d\omega$. The negative correlation between Home productivity and Home consumption

[7] In the remaining part of this paper, I am going to use pricing kernel and stochastic discount factor interchangeably.

[8] Indeed, here I am imposing the marginal utility of tradable goods consumption to be identical across all countries. This assumption is made to save notations. Nothing will change if this assumption is relaxed.

of tradable goods reflects the role of international risk-sharing–when a bad shock hits Home, Home runs a trade deficit to smooth consumption.

Substituting equations (18) and (19) into equation (3) , noting that $\int_{w\in\Omega} A_{\omega t} d\omega / \int_{w\in\Omega} A_{\omega t}{}^{-\gamma} d\omega$ is deterministic due to the law of large numbers when there are no world-wide shocks, yields

$$\frac{C_{t+1}}{C_t} = \exp(g + \phi u_{t+1} + \phi v_{t+1}) \tag{21}$$

where $\phi = \frac{\alpha}{\rho-(\rho-1)\alpha} \in [0, 1]$.

Equation (21) is the equilibrium consumption growth at Home. Foreign consumption growth follows the same equation, except that shocks are Foreign shocks. This equation demonstrates that consumption growth is not perfectly correlated across countries despite complete asset markets. The extent that a country can smooth its aggregate consumption through international financial markets is directly affected by what fraction of goods is tradable. When all goods are non-tradable, i.e. $\alpha = 1$, ϕ is also equal to 1. In this case, the consumption growth is equal to the domestic productivity growth. At the other extreme, when all goods are tradable, $\phi = 0$, the consumption growth rate is a constant regardless of domestic productivity shocks. When α is between 0 and 1, ϕ is also a number between 0 and 1, international risk sharing dampens the effect of domestic shocks, but only to a certain extent. Thus, equation (21) provides a convenient way to gauge consumption correlation across countries and does not have the problem of generating counterfactually high cross-country consumption correlations when α is close to 1.

If we insert equation (20) into equation (8) and recognize that the LOOP holds for tradable goods, the formula for real exchange rate becomes

$$Q_t = (\frac{A_t}{A_t^*})^{\alpha(1+\gamma)} \tag{22}$$

This is also known as the Backus-Smith condition. Using lower-case letters q and e to denote the natural logarithms of real and nominal exchange rates, we have

$$dq_{t+1} = q_{t+1} - q_t = \alpha(1+\gamma)(d\log A_{t+1} - d\log A^*_{t+1}) \quad (23)$$

$$de_{t+1} = e_{t+1} - e_t = dq_{t+1} + \pi_{t+1} - \pi^*_{t+1} \quad (24)$$

where π_{t+1} and π^*_{t+1} are inflation rates at Home and Foreign between period t and period $t+1$, respectively.

3.5 Pricing Kernels and Asset Prices

Given the equilibrium consumption path derived above, we now have well-defined pricing kernels for Home and Foreign. They are respectively

$$m_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\rho} = \beta\exp[-\rho(g + \phi u_{t+1} + \phi v_{t+1})] \quad (25)$$

$$m^*_{t+1} = \beta(\frac{C^*_{t+1}}{C^*_t})^{-\rho} = \beta\exp[-\rho(g + \phi u^*_{t+1} + \phi v^*_{t+1})] \quad (26)$$

No-arbitrage condition requires the following to also be true

$$E_t m_{t+1} R_{t+1} = 1 \quad (27)$$

$$E_t m^*_{t+1} R^*_{t+1} = 1 \quad (28)$$

where R_{t+1} and R^*_{t+1} are the real returns to any assets in Home and Foreign. Equations (25)–(28) will be the core equations for the asset pricing exercise below.

The assets to be studied in this paper can be broadly classified into two categories: stocks and short-term bonds. Stocks include consumption claim, non-tradable consumption claim, dividend claim and levered equity.[9] Short-term bonds include nominal "risk-free" bond and real risk-free bond. Quotation marks refer to the fact that nominal bond is not completely risk-free in real terms as a result of inflation risk. I'll focus on stocks and real risk-free bonds first and postpone discussion on nominal bonds until

[9] I thank John Campbell for suggesting the non-tradable consumption claim.

monetary policy is specified.

3.5.1 Real Risk-Free Bond

Simply rearranging equation (28) gives us the pricing formula for real risk-free bond

$$1 + r^f = \frac{1}{E_t m_{t+1}} \tag{29}$$

For fixed p, substituting equation (25) into above equation yields

$$\begin{aligned} 1 + r^f &= \frac{1}{\beta \exp(-g\rho + \frac{1}{2}\phi^2\rho^2\sigma_u^2)[\exp(-p) + (1-\exp(-p))(1-b)^{-\phi\rho}]} \\ &= 1 - \log(\beta) + g\rho - \frac{1}{2}\phi^2\rho^2\sigma_u^2 - p(1-b)^{-\phi\rho} \end{aligned} \tag{30}$$

where the second equality is true when the period length approaches zero. Foreign has similar equations. Two points to be emphasized here are: 1. When α is equal to 1, we are back to the closed-economy case studied in Barro (2006). Therefore, this model nests Barro (2006) as a special case. 2. Absence of rare disasters, i.e. $p = 0$, the risk-free rate puzzle emerges. For $\rho = 5$ and $g = 2\%$, the risk-free rate will be around 10% per year, not to mention $\rho = 5$ is still too low to generate sizable equity premium. The existence of rare disasters generates strong precautionary saving motive captured by the term $p(1-b)^{-\phi\rho}$, which keeps the risk-free rate low.

3.5.2 Consumption Claim

Following Lucas (1978) and Campbell and Cochrane(1999), stocks can be modeled as a perpetual claim on aggregate consumption stream. Returns to consumption claim is defined as

$$R_{t+1} = \frac{SP_{t+1} + D_{t+1}}{SP_t} \tag{31}$$

where SP is the stock price and D is the dividend, which is identical to aggregate consumption C here. The price/dividend ratio or, equivalently, the price/consumption ratio for consumption claim satisfies

$$\frac{SP_t}{C_t}(p_t) = E_t[m_{t+1}\frac{C_{t+1}}{C_t}[1 + \frac{SP_{t+1}}{C_{t+1}}(p_{t+1})]] \tag{32}$$

The only state variable is the disaster probability p_t, so the price/consumption ratio is a function only of p_t.[10] When p_t is fixed, after substituting the consumption growth from equation (21) and the pricing kernel from equation (25) into equation (32), the following closed-form solution for the price/dividend ratio can be derived

$$\frac{SP}{C}(p) = \frac{\Phi_1}{1 - \Phi_1} \tag{33}$$

and

$$\Phi_1 = \beta \exp(g(1-\rho) + \frac{1}{2}\phi^2(1-\rho)^2\sigma_u^2)[\exp(-p) + (1 - \exp(-p))(1-b)^{\phi(1-\rho)}] \tag{34}$$

Again, this nests Barro (2006) as a special case when $\alpha = 1$ ($\phi = 1$). When p_t is time-varying, only numerical solutions can be obtained. The numerical approach taken is to solve the functional equation (32) on a grid for the state variable p_t, using Monte Carlo method to evaluate the conditional expectation. Given price/consumption ratio as a function of state, I can simulate interesting moments to see how well they match the data.

3.5.3 Non-Tradable Consumption Claim

The formula for pricing non-tradable consumption claim is similar to the one for consumption claim, except that the consumption growth in equation (32) is replaced by the non-tradable consumption growth

$$\frac{SP_t}{C_{Nt}}(p_t) = E_t[m_{t+1}\frac{C_{Nt+1}}{C_{Nt}}[1 + \frac{SP_{t+1}}{C_{Nt+1}}(p_{t+1})]] \tag{35}$$

Closed-form solutions can be obtained when p_t is held fixed, that is

[10] The assumptions of small open economy and i.i.d. normal shocks reduce the number of state variables to only 1. In a two-country version of this model, there are three state variables: Home disaster probability p_t, Foreign disaster probability p_t^* and relative size of Home and Foreign. And if normal shocks are not i.i.d., state variables will be at least 5.

$$\frac{SP}{C_N}(p) = \frac{\Phi_2}{1-\Phi_2} \tag{36}$$

and

$$\Phi_2 = \beta \exp(g(1-\rho) + \frac{1}{2}(1-\rho\phi)^2\sigma_u^2)[\exp(-p) + (1-\exp(-p))(1-b)^{(1-\rho\phi)}] \tag{37}$$

Note that when $\alpha = 1$ ($\phi = 1$), the pricing formula for the non-tradable consumption claim is identical to the pricing formula for the consumption claim. When p_t is time-varying, I can numerically calculate the price/dividend ratio of the non-tradable consumption claim in the same manner as the price/consumption ratio of the consumption claim.

3.5.4 Dividend Claim

The growth rates of stock dividends and consumption are only weakly correlated in U.S. data. Here I treat the dividend claim in the same way as in Campbell and Cochrane(1999) with only one modification. Since consumption growth and output growth are no longer the same process in an open economy model, I let the dividend growth be weakly correlated with output growth instead of with consumption growth during normal times. And when a disaster hits the economy, dividends drop as much as output. Formally, let D denote the level of dividends and d the log of dividends, I specify

$$\Delta d_{t+1} = g + \omega_{t+1} + v_{t+1}; \omega_{t+1} \sim i.i.d.\ N(0, \sigma_\omega^2),\ corr(\omega_{t+1}, u_{t+1}) = \delta \tag{38}$$

where σ_ω^2 is the variance of dividend flow and δ is the correlation between dividend flow and output during normal times.

The price/dividend ratio of dividend claim satisfies

$$\frac{SP_t}{D_t}(p_t) = E_t[m_{t+1}\frac{D_{t+1}}{D_t}[1 + \frac{SP_{t+1}}{D_{t+1}}(p_{t+1})]] \tag{39}$$

Similarly, for fixed p_t, the price/dividend ratio has the following closed-form solution

$$\frac{SP}{D}(p) = \frac{\Phi_3}{1-\Phi_3} \tag{40}$$

where

$$\Phi_3 = \beta \exp(g(1-\rho) + \frac{1}{2}(\rho^2\phi^2\sigma_u^2 - 2\rho\phi\delta\sigma_u\sigma_\omega + \sigma_\omega^2))[\exp(-p) + (1-\exp(-p))(1-b)^{(1-\rho\phi)}] \tag{41}$$

Similar numerical methods as in the case of the consumption claim can be employed to numerically calculate the price/dividend ratio when p_t is time-varying.

3.5.5 Levered Equity

Levered equity is modeled as a combination of short-term debt and equity as in Barro (2006). Since Modigliani-Miller theorem applies, we don't need a separate pricing formula for levered equity. Suppose private bonds B_t are issued at time t at the prevailing nominal interest rate $1 + i_{t+1}$ and the proceeds are given to the equity holders, the return to levered equity is

$$R_{t+1}^{levered} = \frac{SP_{t+1} + D_{t+1} - (1 + i_{t+1} - \pi_{t+1})B_t}{SP_t - B_t} \tag{42}$$

where π_{t+1} is the inflation rate between period t and period $t+1$. Here we are assuming private debt is as safe as "risk-free" nominal government bond.

Equation (42) can be rewritten as

$$R_{t+1}^{levered} = \frac{(\frac{SP_{t+1}}{D_{t+1}} + 1)\frac{D_{t+1}}{D_t} - (1 + i_{t+1} - \pi_{t+1})\frac{B_t}{D_t}}{\frac{SP_t}{D_t} - \frac{B_t}{D_t}} \tag{43}$$

Set $\lambda_t = \frac{B_t}{SP_t}$, which measures the debt/equity ratio, returns to levered equity then become

$$R_{t+1}^{levered} = \frac{(\frac{SP_{t+1}}{D_{t+1}} + 1)\frac{D_{t+1}}{D_t}}{(1-\lambda_t)\frac{SP_t}{D_t}} - \frac{\lambda_t}{1-\lambda_t}(1 + i_{t+1} - \pi_{t+1}) \tag{44}$$

Once we have the price/dividend ratio and the nominal interest rate, returns to levered equity can be easily calculated using equation (44).

3.5.6 Discussion

Before we turn to the nominal side of the model, it is worthwhile to spend some time on certain implications of above formulas. Except the forward discount puzzle, all puzzles this paper tries to address are puzzles about real variables. Since the real side of the economy does not depend on the nominal side, the solutions to those real puzzles do not rely on the nominal side either.

By construction, this model generates a constant short-term real risk-free rate and price/dividend ratios for fixed p_t as shown above. Therefore, any variation in price/dividend ratios and real interest rates in this model is solely from variation in the disaster probability. In another words, changes in asset prices and associated returns in this model are mostly changes in risk premia.[11] In fact, this captures an important idea that is shared by many economists. For example, John Cochrane wrote "...Overall, the new view of finance amounts to a profound change. We have to get used to the fact that most returns and price variation comes from variation in risk premia..." (Cochrane 2001, p. 451). Of course, it is well-known that variation in risk premia is too tiny to explain anything in many models, a natural question is whether variation in risk premia in this model quantitatively matters. It turns out that time-varying disaster probabilities can generate economically meaningful variation in risk premia with a reasonably concave utility function.

A corollary of the above observation is that the UIP condition fails in real terms in this model, i.e., real interest rate differentials are not equal to the expected change of real exchange rate. Intuitively, this is because variation in real interest rates is simply variation in risk premia and has little information about future real exchange rate movements. In fact, the theoretical correlation between real interest rate differentials and the realized change of real exchange rate is 0 conditional on no disasters. To see this, substituting equation (15) into equation (23) yields

$$q_{t+1} - q_t = \alpha(1+\gamma)(u_{t+1} + v_{t+1} - u^*_{t+1} - v^*_{t+1}) \tag{45}$$

Conditional on no disasters, equation (45) becomes

$$q_{t+1} - q_t = \alpha(1+\gamma)(u_{t+1} - u^*_{t+1}) \tag{46}$$

[11] Changes in p_t have negligible effects on average returns.

Since both u_{t+1} and u^*_{t+1} are *i.i.d.* shocks, the change of real exchange rate is by assumption not predictable by any variables observed today, including real interest rate differentials. Running the Fama regression with real variables would get a point estimate of 0. So this model can also deliver a real version of the forward discount puzzle.

As a matter of fact, this model can do just a fine job in terms of real variables without specifying the nominal side–those real puzzles do not depend on the nominal side and the forward discount puzzle has a real counterpart. However, it is still desirable to model the nominal side of this economy explicitly for two important reasons. First, the forward discount puzzle is traditionally a puzzle in terms of nominal variables. Even if it is in fact a puzzle in terms of real variables as some may believe, it is still not clear why it has also robustly been a puzzle in terms of nominal variables ever since 1970's. To see this, the Fama regression in terms of nominal variables (equation (1)) can be rewritten as

$$q_{t+1} - q_t + \pi_{t+1} - \pi^*_{t+1} = \beta_0 + \beta_1[(r_{t+1} - r^*_{t+1}) + E(\pi_{t+1} - \pi^*_{t+1})] + \varepsilon_t$$

where the left-hand side is the change of real exchange rate plus inflation differentials and the right-hand side is the real interest differentials plus expected inflation differentials. Clearly, variation in inflation rates will bias the point estimate of β_1 toward 1 even if running the Fama regression in terms of real variables yields $\beta_1 \neq 1$ or even $\beta_1 < 0$. This problem will be especially severe when inflation rates are volatile, which were probably true for industrial countries before mid-1980's.[12] So a forward discount puzzle in terms of real variables does not automatically guarantee a forward discount puzzle in terms of nominal variables. Therefore, it is not trivial to model the nominal side of the economy explicitly. Second, more fundamentally, nominal exchange rates and real exchange rates are two very different things. Just imagine in a world where the LOOP always holds so real exchange rates are always 1, the nominal exchange rates can still be anything depending on the fundamentals such as real consumption and money supply now and in the future. Nominal exchange rates are more like asset prices that are determined largely by future fundamentals, while real exchange rates are the relative prices clearing current international good markets.[13] Although real exchange rates move very closely with nominal exchange rates in the data presumably due to sticky prices, the theoretical

[12] For example, see World Economic Outlook (IMF 2006).
[13] See Devereux and Engel (2007) for more discussions on this point.

underpinnings of these two variables are quite different. We certainly do not want to rush to any conclusions about nominal exchange rates by looking at real exchange rates only. The setup of this paper pushes the difference between real and nominal exchange rates to the extreme–real exchange rates are determined entirely by current consumption (see equation (22) or (45)) and nominal exchange rates are the prices of currencies. As it will become clear later on, this distinction makes difference.

Readers who are not interested in the nominal side can skip the next section and go directly to the calibration and simulation part. As discussed above, except the forward discount puzzle, other results of this paper do not particularly rely on the nominal side.

3.6 Inflation and Nominal Interest Rate

3.6.1 Money as Stock

The first order condition for money demand at Home from individual maximization problem is

$$1 - \chi C_t^{\rho}[\frac{M_t}{P_t}]^{-\epsilon} = E_t m_{t+1} \frac{P_t}{P_{t+1}} \tag{47}$$

After ruling out speculative bubbles, money can be treated as a long-term asset. The trick is to rewrite equation (47) in the following recursive form

$$\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\rho}{\epsilon}}}(p_t) = 1 + E_t[m_{t+1}(\frac{C_{t+1}}{C_t})^{\frac{\rho}{\epsilon}} \frac{M_t}{M_{t+1}} \frac{M_{t+1}}{\chi^{\frac{1}{\epsilon}} P_{t+1} C_{t+1}^{\frac{\rho}{\epsilon}}}(p_{t+1})] \tag{48}$$

where $\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\rho}{\epsilon}}}$ is the consumption-adjusted real money balance that is a function only of p_t.[14]

By writing the money demand function in this way, money is just like stocks.[15] The "dividend" to such a stock is $(\frac{C_{t+1}}{C_t})^{\frac{\rho}{\epsilon}} \frac{M_t}{M_{t+1}}$, which is the reciprocal of consumption-adjusted money growth. When consumption growth is high, the "dividend" is also high because money is more valuable now in facilitating transactions. However, when money growth is high, the "dividend" becomes less valuable as a result of increased inflation. Treating money as stock provides a useful way to think about money both conceptually and technically. Conceptually, this explains why the fiscalist version of this model turns out to be isomorphic to the monetarist version. In the fiscalist version, money(or nominal bond)

[14] Obstfeld and Rogoff (2002) use similar tricks to yield closed-form solution when $\epsilon = 1$.

[15] Cochrane (2005) argues that money is stock from a fiscalist perspective.

is modeled as a claim on future government primary surplus. As long as the dividend streams are the same, whether we are living in a monetarist world or a fiscalist world or somewhere in between makes no difference for the price level and inflation, which will always be the same. Technically, I can solve out the consumption-adjusted real money balance in the same fashion as the price/dividend ratio for stocks when p_t is fixed and the same numerical technique applies when p_t is time-varying. Once $\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\varrho}{\epsilon}}}(p_t)$ is known, inflation and nominal interest rates can be easily calculated with the following formulas

$$\pi_{t+1} = \frac{\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\varrho}{\epsilon}}}(p_t)}{\frac{M_{t+1}}{\chi^{\frac{1}{\epsilon}} P_{t+1} C_{t+1}^{\frac{\varrho}{\epsilon}}}(p_{t+1})} \frac{M_{t+1}}{M_t} (\frac{C_{t+1}}{C_t})^{-\frac{\varrho}{\epsilon}} \tag{49}$$

and

$$1 + i_{t+1} = \frac{1}{E_t m_{t+1} \frac{1}{\pi_{t+1}}} \tag{50}$$

3.6.2 Monetary Policy

Monetary policy must be specified before price level, inflation and nominal exchange rates can be discussed. Following the tradition of monetarism, monetary policy is a state-contingent plan of aggregate money supply.[16] Money does not affect real output and there is no dynamic-inconsistency problem in this economy. Therefore, what the optimal monetary policy is is of no concern. Also, the focus of this paper is the asset pricing implications of rare disasters, so monetary shocks will be completely ignored in the discussion below.[17]

It turns out that what a monetary authority commits to doing during a disaster has significant effects on price level and inflation. The reason is that the demand for money depends heavily on people's expectations about future money supply and consumption growth, and such expectations fall disproportionately on what the central bank does during a disaster, which is the state people care about the most.[18]

Two monetary policies are considered. One is what I call the "Naive Monetary Policy" and the

[16] In the fiscalist version, the price level is determined by fiscal policy. "Monetary policy" in that economy will be a state-contingent plan of government primary surplus.

[17] In fact, monetary shocks are similar to pure noise in this model since money is neutral by assumption and there is no propagation mechanism for pure monetary shocks to have any lasting effects on the economy and asset prices.

[18] Technically speaking, the stochastic discount factor is very high for the state of disasters.

other is dubbed "Well-anchored Inflation Expectation". Both policies can achieve perfect price stability when p_t is fixed but have very different implications on inflation volatility when p_t is time-varying. It makes no difference whether trend inflation is allowed or not.[19] Also in order to take into account the fact that hyperinflation may sometimes occur during or immediately after a disaster, for instance the German hyperinflation after World War I, I allow for the possibility of such hyperinflation scenarios during disasters. Another way to think about the hyperinflation scenarios is that government may partially default on its debt during a disaster.

The "Naive Monetary Policy" is specified as following

$$M_{t+1} = \kappa C_{t+1}^{\frac{\varrho}{\epsilon}} \text{ during normal time} \tag{51}$$

$$M_{t+1} = \kappa F C_{t+1}^{\frac{\varrho}{\epsilon}},\ F > 1,\ \text{with prob. } q \text{ during a disaster} \tag{52}$$

$$M_{t+1} = \kappa C_{t+1}^{\frac{\varrho}{\epsilon}},\ \text{with prob. } 1-q \text{ during a disaster} \tag{53}$$

where κ is a constant. Essentially, this is a monetary policy that accommodates economic growth by keeping up money supply with consumption growth. But during a disaster, with probability q hyperinflation or partial default may happen. F in equation (52) measures the size of hyperinflation or partial default. When p_t is fixed and suppose $F = \frac{1}{1-b}$, it is straightforward to show that the consumption-adjusted real money balance $\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\varrho}{\epsilon}}}$ and the price level P_t satisfy

$$\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\varrho}{\epsilon}}}(p) = \frac{1}{1-\Xi_1} \tag{54}$$

and

$$P_t = \kappa(1-\Xi_1) \tag{55}$$

where $\Xi_1 = \beta \exp(-g\rho + \frac{1}{2}\rho^2\phi^2\sigma_u^2)[(1-p) + pq(1-b)^{1-\rho\phi} + p(1-q)(1-b)^{-\rho\phi}]$.[20]

From equations (54) and (55), it is evident that complete price stability is achieved under this policy as long as p_t is fixed. However, it also becomes clear that this may not be the best monetary

[19] Certainly, from a welfare point of view, a negative inflation rate may be more desirable when money is neutral, see Friedman (1969).

[20] Barro (2006) makes the same assumption that $F = \frac{1}{1-b}$.

policy a central bank wants to adopt when p_t is time-varying as far as price stability is concerned. Ignoring the possibility of hyperinflation for a moment, i.e. $q = 0$, the volatility of the price level and the inflation rate depends crucially on the term $p(1-q)(1-b)^{-\rho\phi}$ in Ξ_1, which is quite volatile even for modest movements in p because $(1-b)^{-\rho\phi}$ is quite a large number.[21] The fact that a central bank always commits to contracting the money supply as much as the whole economy does during a disaster regardless of the probability of such events turns out to be somewhat "destabilizing". People care very much about the bad state. With the probability of bad states changing over time, people's expectations also change quite a bit. In this sense, the inflation expectation is not well-anchored under this simple monetary policy.

A simple way out is to make the monetary policy conditional on the disaster probability, the particular monetary policy I considered is

$$M_{t+1} = \kappa C_{t+1}^{\frac{\varrho}{\epsilon}} \text{ during normal time} \tag{56}$$

$$M_{t+1} = \kappa F C_{t+1}^{\frac{\varrho}{\epsilon}},\ F > 1,\ \text{with prob. } q \text{ during a disaster} \tag{57}$$

$$M_{t+1} = \kappa \frac{p_{t+1}}{p_{bench}} C_{t+1}^{\frac{\varrho}{\epsilon}},\ \text{with prob. } 1-q \text{ during a disaster} \tag{58}$$

The only difference from the naive monetary policy is that a central bank is committed to a probability-adjusted money supply rule should a disaster occur next period. Given that disasters rarely happen, this is almost like an off-equilibrium path commitment that stabilizes expectations. This monetary policy is also equivalent to commitment to tighter money supply when inflation expectations are high. Under this policy, for fixed p_t we have

$$\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\varrho}{\epsilon}}}(p) = \frac{1}{1-\Xi_2} \tag{59}$$

and

$$P_t = \kappa(1-\Xi_2) \tag{60}$$

where $\Xi_2 = \beta \exp(-g\rho + \frac{1}{2}\rho^2\phi^2\sigma_u^2)[(1-p) + pq(1-b)^{1-\rho\phi} + p_{bench}(1-q)(1-b)^{-\rho\phi}]$. Again,

[21] In the benchmark calibration, $(1-b)^{-\rho\phi} = 8.8$.

complete price stability is achieved when p_t is fixed. Note that $p(1-q)(1-b)^{-\rho\phi}$ in Ξ_1 is replaced by $p_{bench}(1-q)(1-b)^{-\rho\phi}$ in Ξ_2, thus changes in disaster probability do not cause very volatile inflation expectations. By committing to a probability-adjusted money supply rule during a disaster, inflation expectations are well-anchored under this policy. With well-defined monetary policies, I can use equations (49) and (50) together with either equations (51)–(53) or equations (56)–(58) to calculate the price level, inflation and the nominal interest rate. Nominal exchange rates can also be calculated using equation (24).

This concludes specifications and analytical solutions of the model. The next step is to calibrate the model and to simulate it to see how well it can match the data.

4 Calibration and Simulation

Table 1 lists all parameter values used in the benchmark calibration. One period in the calibration is one year. A wide range of values have been assumed for the discount factor β in the literature.[22] I set $\beta = .97$, which is a conventional value. The consumption growth rate $g = .0189$ and standard deviation $\sigma_u = .015$ match the postwar per capita nondurables and services consumption data in the U.S. described in Campbell and Cochrane(1999). The drop of output during a disaster b is assumed to be fixed at 42%. It would make no difference if b is a random variable with a time-invariant mean. A 42% drop in output seems a bit high at first glance. But first of all, this is by no means impossible. Countries like South Korea even experienced a 60% drop in GDP during WWII. The mean of the adjusted contraction sizes over 60 disasters is 35% in Barro (2006).[23] In the real world capital markets are not nearly complete, so that the adverse effects of a slightly milder disaster on individual investors could be comparable to those of more severe disasters in a world with perfect capital markets as I assume. In the model, a 42% percent drop in output leads to "only" a 33% drop in consumption under the benchmark calibration. From this perspective, a 42% drop in output does not seem to be an implausible assumption. Following Barro (2006), the size of hyperinflation (or the size of partial default) are assumed to be the same as economic disaster itself, so $F = \frac{1}{1-b}$ and the probability of hyperinflation conditional on disaster q is set to 0.4. Given that the coefficient of relative risk aversion

[22] Bansal and Yaron (2004) assume $\beta = .976$, Campbell and Cochrane (1999) set β to be 0.89. According to the structural estimation in Gourinchas and Parker (2002), β can be anywhere between .99 and .92 in the baseline estimation.

[23] The adjusted contraction size is the actual fall in GDP adjusted for trend growth.

ρ can be anywhere between 1 and 10 in the macroeconomics and finance literature, I choose $\rho = 5$.[24] ρ needs to be reasonably high to account for both high equity premia and exchange rate volatility. The share of non-tradable goods α is 0.95, which is meant to capture the notion that consumption correlation is low across countries. $\alpha = 0.95$ may be too high if we take "non-tradable" literally. But since markets are assumed to be complete, $\rho = 5$ and $\alpha = 0.95$ together imply $\phi = 0.8$, which means that 20% of the idiosyncratic shocks are eliminated through international risk-sharing. This is quite substantial. In reality, the share of non-tradable goods is lower but markets are not complete. In terms of delivering the consumption pattern mimicking the data, complete markets plus a high non-tradable share do just the job. This is more a modeling strategy rather than reflecting the reality. In fact, similar assumptions have been made in Chari et al. (2002) and Steinsson (2007).[25] The benchmark disaster probability p_{bench} is 0.017, which matches 60 disastrous events for 35 countries in the 20th century documented in Barro (2006). η and σ_ϵ govern the evolution of the disaster probability. In order to generate volatile and persistent price/dividend ratio, the probability of disasters must be quite persistent as well, therefore I chose $\eta = 0.95$ and $\sigma_\epsilon = 0.0025$. δ and σ_ω are the covariance between consumption growth and dividend growth and the standard deviation of dividend growth during normal times. The correlation between consumption growth and dividend growth is tricky to measure and the point estimates are subject to large sampling error as discussed in Campbell and Cochrane (1999). But it turns out that this number does not have a significant effect on stock prices. Therefore, I simply use $\delta = 0.2$ and $\sigma_\omega = 11.2\%$ in Campbell and Cochrane (1999). The debt/equity ratio λ for levered equity is set to be 0.5 when $p_t = p_{bench}$. This is roughly consistent with recent debt-equity ratios for the U.S. non-financial corporate sector according to the Federal Reserve's Flow-of-Funds Accounts.

[INSERT TABLE 1 HERE]

I first calculate asset prices using the formulas and numerical methods described in the previous section. Then I simulate 1,000,000 years of artificial data for both Home and Foreign, assuming that disasters never materialize. Obviously, it does not seem remotely reasonable to assume that disasters

[24] Campbell and Cochrane (1999) set $\rho = 2$, Barro(2006) uses $\rho = 4$, Chari, Kehoe and McGranttan (2002) use $\rho = 5$, Bansal and Yaron (2004) choose $\rho = 7.5$ and $\rho = 10$ and Mehra and Prescott (1985) argue that $\rho \leq 10$ is acceptable.

[25] Chari et al. (2002) use $\alpha_1 = .94$ in their benchmark calibration, where α_1 measures degree of home bias. Steinsson (2007) uses similar parameter value.

never happen over 1,000,000 years, but the objective here is to calculate the population values for a variety of statistics conditional on no disasters. For this purpose, I can simulate either 50 years of data 20,000 times or 1,000,000 years of data only once and the latter is adopted. In section 6, I will allow for actual disasters when doing a robustness check. Monetary policy is assumed to be the "Well-anchored Inflation Expectation" one in the previous section.

Table 2 reports the simulation results. For comparison purposes, the last two columns are the historical statistics, which are from Campbell and Cochrane (1999), Barro (2006) and my own calculation.[26]

The first two rows are the mean and standard deviation of returns to real risk-free bonds, which do not exist in reality.[27] Despite the fact that people have very low IES and the economy is growing, returns to real risk-free bond, which I call the true risk-free rate, remain low when rare disasters are possible. The next two rows are the mean and standard deviation of real returns to nominal risk-free bonds. The mean return remains quite low but somewhat higher than the true risk free rate due to the inflation risk. The standard deviation is close to its historical level, so the time-varying disaster probability does not generate too volatile a risk-free rate. The next three rows are excess returns, standard deviations of excess returns and the implied Sharpe ratios for equities. Equity premia fall into the right ballpark of historical average for all four equities considered. Volatility of excess returns for the dividend claim matches the data quite well. However, excess returns to the consumption claim and the non-tradable consumption claim are not as volatile as in the data. This is because the dividend streams to these two claims are relatively smooth. The implied Sharpe ratios are close enough to their historical counterparts for the dividend claim and levered equity. The Sharpe ratios for the consumption claim and the non-tradable consumption claim are a little bit high due to relatively smooth dividend streams. The last two rows in the asset price panel are price/dividend ratios and their volatility. We can see that equity prices in the model are as volatile as they are in the data and the level of price/dividend ratio also fits the historical average quite well.

[26] The historical statistics for stocks in the U.S. are from Campbell and Cochrane (1999), where the long sample covers the period of 1871-1993 and the post-war sample covers that of 1947-1995. The historical statistics for real returns to the U.S. Treasury bill are from Barro (2006), wherein the long sample covers the period of 1880-2004 and the post-war sample covers that of 1954-2004. The statistics for exchange rates are calculated by the author, where the time-frame is from 1973 to 2007.

[27] TIPS may be very close to real risk-free bond.

For exchange rates, this model exhibits volatile exchange rates, both nominal and real. The standard deviations are 8.6% and 9%, respectively, close to their historical counterparts.[28]

Another question is whether this model can generate the forward discount puzzle and the answer is yes. The bottom row in table 2 reports the mean and standard error of β_1 of the Fama regression (equation (1)) using the simulated data. I simulate 1000 years of data 1000 times and the mean and the standard error reported are the mean and standard error of the point estimates of β_1 from these 1000 simulations. β_1 is obviously different from 1, which is what the uncovered interest parity (UIP) condition predicts. And in the majority of simulations, β_1 has a negative point estimate, although with large standard errors $\beta_1 = 0$ cannot be ruled out statistically. All these results are consistent with a large empirical literature on the forward discount puzzle, where the consensus view is the Fama regression and all its variations usually yield a negative point estimate of β_1. In most cases, β_1 is statistically different from 1 but it becomes less clear whether β_1 is statistically different from 0.[29]

[INSERT TABLE 2 HERE]

Table 3 and 4 report autocorrelations and cross-correlations from the simulated data and the results are also quite encouraging. Price/dividend ratios are persistent. They are a little bit more persistent than the data suggests but not too far off from it. Excess returns display small negative autocorrelations as in the data (Fama and French 1988, Poterba and Summers 1988).

The cross-correlation between the price/dividend ratio and subsequent excess returns verifies that price/dividend ratio forecasts long-horizon returns with the correct sign: high prices forecasts low returns. Note that the dividend flow in this model by assumption is not forecastable, so the forecastability of future returns is completely due to disaster probability following a mean reverting process. The correlation is slightly smaller for dividend claim since its returns are nosier. This model also has the "leverage effect" that Black (1976), Schwert (1989), Nelson (1991) and many others document-low price/dividend ratio signals high volatility for several years ahead. The cross-correlations between the price/dividend ratio and absolute value of subsequent returns in table 4 demonstrate this effect.

[INSERT TABLE 3 and 4 HERE]

[28] The historical statistics are calculated from the trade-weighted exchange rates of US, Japan, UK and the Euro area constructed by the Bank for International Settlements(BIS).

[29] See Froot and Thaler (1990) and Engel (1996) for review of relevant empirical studies, see also Backus, Foresi and Telmer (2001).

So overall this model can replicate many salient features in the stock price and exchange rate data. By construction, this model has low consumption correlations across countries and non-predictable dividend flows, and it can still generate very volatile stock prices, quite volatile exchange rates, high and volatile equity premia, low and smooth risk-free rates, long-term predictability of excess returns to stocks and the forward discount puzzle. As claimed, the equity premium puzzle, the risk-free rate puzzle, the forward discount puzzle, excess volatility and the volatility mismatch puzzle are explained within this model. But what is the economics behind these results?

5 Model Intuition

The intuition of why rare disasters can help explain the equity premium puzzle and the risk-free rate puzzle is already well-developed in Rietz (1988) and Barro (2006). On the one hand, rare disasters make stocks much riskier than they are in a standard RBC model calibrated only to normal consumption shocks, and thus substantially increase the "premium" people demand for holding stocks. On the other hand a strong precautionary saving motive due to the existence of disasters keeps the risk-free rate low, even though the IES is by assumption relatively low($IES = 0.2$ in the benchmark calibration).

The extra explanatory power of this model mainly comes from the time-varying probability of disasters. Since stocks are modeled as perpetual claims on certain dividend flows, a small change to the disaster probability could cause large swings in stock prices if such a change is not too temporary. The reason is that such a change considerably increases the overall probability of being hit by a large disaster in the future. We can see this point most clearly from equation (33), (36) or (39), where the change in probability is treated as permanent. Take equation (33) for instance, the price/dividend ratio of consumption claim can be thought of as a summation over the infinite series $\Phi_1, \Phi_1^2, \Phi_1^3, ...\Phi_1^n, ...$ A small change in p causes some change in Φ_1 and such a change eventually translates into big change in the overall price/dividend ratio through the infinite summation. Table 5 calculates the corresponding price/dividend ratios for different disaster probabilities p under the benchmark calibration. It is evident that seemingly small changes in p do cause large changes in stock prices.

It may have been a concern that time-varying disaster probability may also generate very volatile real interest rate which would be counterfactual, but this turns out not to be the case. In fact, short-term real interest rate only depends on the expected consumption growth between this period and the next period and disaster probabilities beyond next period do not matter at all. Therefore a change in p does not have the cumulative effect on the real interest rate as it does on stock prices.

[INSERT TABLE 5 HERE]

High persistence in stock prices, negatively autocorrelated excess returns, forecastability of long-horizon returns and the leverage effect are all due to the slow mean-reverting p_t. Mechanically, the mechanism is quite similar to Campbell and Cochrane (1999), in which the slow mean-reverting process is not the disaster probability but the "external habit", although the underlying economics is fundamentally different. As discussed in the previous section, all stock prices are functions only of the disaster probability. With a persistent disaster probability, we automatically have persistent stock prices. But since disaster probability is mean-reverting, eventually everything reverts to its mean in the absence of any additional shocks. Above average excess returns today imply lower excess returns tomorrow, and thus excess returns are negatively autocorrelated. Above average stock price signals lower prices in the future and therefore high stock price predicts low returns. The leverage effect can be directly seen from equation (31), for given volatility of $SP_{t+1} + D_{t+1}$, a lower SP_t implies higher volatility of future returns.

This model generates volatile exchange rates through the combination of a high coefficient of relative risk aversion and a low international consumption correlation. A high coefficient of relative risk aversion is also needed for generating a high equity premium. According to equation (23), the volatility of exchange rates is the volatility of relative productivity growth between Home and Foreign multiplied by $\alpha(1+\gamma)$, which is an increasing function of both α and ρ, the fraction of goods that is non-tradable and the coefficient of relative risk aversion. Here α controls the consumption correlation across countries. When $\alpha = 1$, i.e. all goods are non-tradable, consumption is correlated across countries only if productivity is correlated. But when $\alpha = 0$, consumption is always perfectly correlated across countries regardless of idiosyncratic shocks. As consumption becomes more and more correlated across countries, exchange rates become less and less volatile. When $\alpha = 0$, the law of one price always holds and there will be no exchange rate volatility whatsoever. So in order to generate volatile exchange rates, consumption correlation must be quite low, i.e. we need a high α. We also need a reasonably high ρ to generate volatile exchange rates. With a high ρ, a small difference in the consumption growth rate can cause large exchange rate fluctuations. This is exactly the point made in Chari, Kehoe and

McGrattan (2002).

It is important to point out that the volatility of stock prices and the volatility of exchange rates are driven by completely different forces. Volatile stock prices are mainly due to time-varying disaster probability while volatile exchange rates are entirely due to consumption differentials between Home and Foreign. This explains why the model survives the Brandt, Cochrane and Santa-Clara (2006) critique or the volatility mismatch puzzle. Because disaster probability has nothing to do with consumption in this model, we can have volatile stock prices on the one hand and relatively less volatile exchange rates on the other hand while maintaining a low cross-country consumption correlation.

The reason that this model also generates the forward discount puzzle is the so-called "safe-haven" effect.[30] When Home disaster probability increases, Home demand for safe assets increases. In this model, the two safest assets for the Home representative agent are Home nominal risk-free bond and Home currency, both of which are subject only to inflation risk. In terms of riskiness, nominal bonds and currency are the same. An increased demand for Home bonds leads to lower interest rate. We can see this from equation (30), where interest rate is a decreasing function of p. Meanwhile, an increased demand for Home currency causes an immediate Home appreciation. More specifically, according to equation (59), an increase in Home disaster probability lowers Ξ_2 and therefore increases $\frac{M_t}{\chi^{\frac{1}{\epsilon}} P_t C_t^{\frac{\rho}{\epsilon}}}$, i.e. more demand for real money balances. Since money supply is fixed at $\frac{M_t}{C_t^{\frac{\rho}{\epsilon}}} = \kappa$, the only way to clear the money market is to have the Home price level P_t fall immediately. For a given real exchange rate, which is determined by consumption differentials, a drop in the Home price level corresponds to an appreciation of the Home currency. But since disaster probability is mean-reverting, the appreciating Home currency will on average depreciate next period. Thus, on the one hand we have a lowered Home interest rate, and on the other hand Home currency depreciates on average in the next period. This creates a negative correlation between the Home interest rate and the change in the exchange rate–precisely the forward discount puzzle–a currency that pays a lower interest rate on average depreciates. Graph 1 shows the impulse-response of a sudden drop of Home disaster probability p, which is the opposite case of that just described. The upper panel is the disaster probability, which drops initially and gradually reverts to its mean. The middle panel is the exchange rate, which experiences a sudden depreciation followed by a steady appreciation. The lower panel is the Home nominal interest rate,

[30] Obstfeld and Rogoff (2002) briefly discuss the safe-haven effect. However, they do not discuss what might cause this effect and whether it matters quantitatively.

which increases initially and then declines steadily. This verifies the intuition just described. This graph also reminds us of the celebrated Dornbusch overshooting model, where the UIP condition is assumed to be true. In that model, a drop in Home interest rate is associated with a large initial Home depreciation and a steady appreciation subsequently. In this model, exchange rates also overshoot, i.e. a large initial depreciation followed by a gradual appreciation. But the initial response of the interest rate is just the opposite.[31] It is noteworthy that this overshooting feature is not present in real exchange rates since real exchange rates always follow a unit root process during normal times regardless of changes in the disaster probability (see equation (46)). This highlights the distinction between nominal exchange rates and real exchange rates. Nominal exchange rates are asset prices that respond to news about future fundamentals, while real exchange rates are just relative prices that serve to clear current international good markets. Foreign bonds and currency are not a good hedge against Home disasters because of the adverse exchange rate movements during a disaster. Real returns to Foreign bonds and currency will be extremely low during a Home disaster as the Home exchange rate appreciates by a lot. After all, holding Foreign bonds is just like holding claims on Foreign tradable goods, while in a Home disaster what is most needed is the Home non-tradable goods.

[INSERT GRAPH 1 HERE]

6 Robustness Check

It may be a concern as to how robust the results are in the previous section. How do the moments of simulated data look like if disasters are allowed to occur in the simulation? What if a government never creates hyperinflation even during a disaster? What if a government adopts the naive monetary policy? How sensitive are the results to different parameter values?

Table 6 reports the summary statistics of key moments from the simulated data under two different experiments. The first experiment is that I allow for 17 disasters hitting Home over a 1000-year time frame, or equivalently 17,000 disasters over 1,000,000 years. This experiment is designed to see whether

[31] I thank Eric Van Wincoop and John Rogers for raising this point. The differences between this model and the overshooting model are a. the UIP condition is not taken as given in this model and b. shocks in this model are change in disaster probability and shocks in the overshooting model are monetary shocks. Also, "delayed overshooting" documented in Eichenbaum and Evans (1995) is consistent with the forward discount puzzle. However, delayed overshooting is not a robust finding among non-U.S. G-7 countries, see Kim and Roubini (2000). Given that the forward discount puzzle is a prevalent puzzle among many different country pairs, delayed overshooting is probably not responsible for the forward discount puzzle.

previous results are purely driven by the small sample problem. The column under "actual disasters" reports the simulation results. As we can see, nothing changes significantly. Allowing actual disasters to occur lowers returns to equities somewhat but not by much. Equity premium is still high. This highlights that the mechanism that generates a high equity premium and a low risk-free rate in the rare-disaster approach is not disasters per se, but rather the fear of potential disasters. After all, disasters rarely occur and therefore allowing for actual disasters has little effects on returns to stocks. Exchange rates become more volatile than the benchmark case due to large exchange rate movements during disasters. With noisier exchange rates, the Fama regression still yields on average negative point estimates, but with a larger standard error. This experiment confirms that this paper is *not* a peso-problem-type explanation of the equity premium puzzle and the forward discount puzzle. In other words, equity premia and returns from betting against the UIP condition are not due to the small sample problem. Even with an infinite sample that includes disaster periods, average returns to stocks will still be higher than returns to risk-free bonds and low interest rate currencies will still on average depreciate.

The second experiment I consider is to have central banks adopt the naive monetary policy. The column under "naive monetary policy" reports simulation results. As expected, inflation becomes more volatile under the naive monetary policy. Inflation volatility also causes nominal exchange rates to be significantly more volatile than real exchange rates, which have the same volatility as in the benchmark case. Equity premium for stocks increases by a little bit. It is interesting to note that volatility of excess returns is actually lower in this case. This is quite understandable since there are a lot more comovements between stock prices and inflation now, both of which are driven by the disaster probability. This also explains why excess returns to levered equity are lower. The Fama regression still yields negative point estimates on average. Due to increased variation in interest rates, the standard error of β_1 is even smaller in this case.

In summary, these two alternative specifications do not change the main results obtained in the benchmark calibration by very much.

[INSERT TABLE 6 HERE]

Table 7 reports simulation results under different parameter values. A lower ρ means that people are less risk-averse. Therefore, disasters are not as fearful as before. A lower ρ in theory should have two offsetting effects on the real interest rate. Decreased precautionary saving motive causes real interest rates to rise while a higher IES causes real interest rates to fall. However, with time-separable utility the former always dominates the latter and that's why we see a rise in the interest rate. A similar reasoning is behind the drop in the mean price/dividend ratio. The equity premium drops as expected since people are less risk-averse. The volatility of stock prices drops simply because people are less responsive to a change in disaster probability. Exchange rate volatility also drops mechanically with a lower ρ. The Fama regression still has negative point estimates but with a larger standard error.

When hyperinflation never occurs during a disaster, i.e. $q = 0$, nominal bond becomes a safer asset and hence real returns to nominal bonds drop and equity premium increases. Other than this, other statistics look very similar to the benchmark case.

A higher consumption growth rate g raises both returns to bonds and equities without changing the equity premium by very much. For a given IES, a higher consumption growth rate corresponding to a higher interest rate is exactly what we should expect. This can be seen from the $g\rho$ term in equation (30). Exchange rates are not affected by trend growth rate and the forward discount puzzle is still there.

The main effect of having a less persistent process of disaster probability, i.e. a lower η, is that stock prices and returns become less volatile. The reason is quite straightforward–if the change in disaster probability is more temporary, then it would have less of an impact on asset prices. Equity premium barely changes but the volatility of excess returns drops significantly. However, a more mean-reverting process of p_t makes the forward discount puzzle more pronounced as the point estimate of β_1 in the Fama regression becomes more negative. This is because the subsequent appreciation following the initial depreciation becomes stronger when p_t reverts to its mean faster after the initial drop.

When α drops from .95 to .80, it has a significant impact on the quantitative performance of this model. α, the share of non-tradable consumption, governs how much idiosyncratic risks can be smoothed away through international risk-sharing. When $\alpha = 0.95$, only 20% idiosyncratic risks can be shared internationally. But when $\alpha = 0.8$, this number increases to 56%. Now, disasters are not that disastrous as far as consumption is concerned. It is not surprising that the equity premium puzzle reemerges, the risk-free rate rises and $\beta_1 = 1$ cannot be rejected statistically in the Fama regression. But from a different perspective, this suggests that welfare gains from international risk-sharing may be much larger in this model than in a standard RBC model which fails to consider rare but severe disasters. A thorough welfare analysis is beyond the scope of this paper. But according to the calculation in Barro (2007), a country may be willing to lower real GDP by 20% to eliminate all disaster risks. If better international risk-sharing helps to partially eliminate disaster risks, the welfare gain of international risk-sharing could be substantial as well.

With a higher β, people become more patient. They are willing to save more and discount the future by less. This implies a lower interest rate and a higher price/dividend ratio. The last column of table 7 confirms this intuition. A change in β does not have a noticeable effect on other variables.

Overall, the qualitative implications of this model do not change much under different parameter values and under alternative specifications. This suggests that the results in the benchmark calibration are quite robust. The quantitative implications do change somewhat with respect to certain parameters, but this is common among calibration exercises and the economics behind such changes makes sense.

[INSERT TABLE 7 HERE]

7 Epstein-Zin-Weil Preferences

Despite its simplicity, an undesirable feature of the time-addititive isoelastic preferences is that the intertemporal elasticity of substitution is simply the reciprocal of the coefficient of relative risk-aversion. From a theoretical point of view, there is no obvious reason to believe why this is necessarily the case. Furthermore, this feature generates a counterintuitive and probably counterfactual asset pricing implication in the model above. We can see it most clearly in table 7, where an increase in disaster probability p causes the stock prices to rise not to fall. The reason is quite simple: an increase in p has two offsetting effects–the precautionary saving effect that increases demand for the risky assets and the substitution effect that lowers demand for the risky assets. But when the IES is less than 1, which is true for a risk-averse agent with time-separable utility, the precautionary saving motive always dominates. With fixed supply of assets, the price of risky assets must increase to clear the market. That's why the stock prices counterintuitively rise when disasters become more likely.

One way to get around with this problem is to use the recursive preferences introduced by Epstein and Zin (1989) and Weil (1989). The Epstein-Zin-Weil preferences allow the coefficient of risk-aversion to differ from the inverse of IES. From previous discussions, we know that it is possible to make asset prices fall when p rises by having $IES > 1$. The problem is whether other results developed in the previous sections still hold under such preferences. A thorough study of the asset pricing implications of rare disasters under the Epstein-Zin-Weil preferences is the subject of another ongoing research. Here I am just laying out a very simple model to illustrate the intuition. The findings actually are quite encouraging: other than the fact that the volatilities of stock prices and exchange rates are at odds with the data, which may be an artifact of some oversimplified assumptions, all previous results carry through. Furthermore, the Epstein-Zin-Weil preferences also shed some light on the relationship between stock markets and exchange rates and the predictions are consistent with empirical findings in Hau and Rey (2006).

Following Bansal and Shaliastovich (2007) and Colacito and Groce (2005), I assume that the asset markets are complete and each country consumes only its own endowment, which is the limiting case when we let α approach 1 in the previous model. For simplicity, I also ignore the nominal side of this economy. The Home representative has the following utility function

$$U(C_t, E_t V_{t+1}) = \left[(1-\beta)C_t^{1-\theta} + \beta(E_t V_{t+1})^{\frac{1-\theta}{1-\rho}}\right]^{\frac{1-\rho}{1-\theta}} \tag{61}$$

where β is the subjective discount factor, $\frac{1}{\theta}$ measures the constant intertemporal elasticity of substitution and ρ still parameterizes the coefficient of relative risk-aversion.

As shown in Epstein and Zin (1989) and Weil (1989), the stochastic discount factor for above preferences is given by

$$m_{t+1} = \left[\beta(\frac{C_{t+1}}{C_t})^{-\theta}\right]^{\frac{1-\rho}{1-\theta}} R_{c,t+1}^{\frac{1-\rho}{1-\theta}-1} \tag{62}$$

where $R_{c,t+1}$ is the gross return to the consumption claim that delivers the aggregate consumption flow.

The Foreign representative agent's stochastic discount factor is similarly defined

$$m^*_{t+1} = \left[\beta(\frac{C^*_{t+1}}{C^*_t})^{-\theta}\right]^{\frac{1-\rho}{1-\theta}} R_{c,t+1}^{*\frac{1-\rho}{1-\theta}-1} \tag{63}$$

With m_{t+1}, m^*_{t+1} and the Euler equations, we can price any assets in Home and Foreign. As emphasized by Epstein and Zin (1989), asset prices now depend not only on the covariance between returns and consumption growth, but also on the covariance between returns of that asset and returns to the consumption claim. A two-step procedure is adopted to compute asset prices. First, after some simple manipulations of the Euler equation, we have the following pricing formula for the consumption claim

$$\left[\frac{SP_t}{C_t}(p_t)\right]^{\frac{1-\rho}{1-\theta}} = E_t[\beta^{\frac{1-\rho}{1-\theta}}(\frac{C_{t+1}}{C_t})^{1-\rho}[1+\frac{SP_{t+1}}{C_{t+1}}(p_{t+1})]^{\frac{1-\rho}{1-\theta}}] \tag{64}$$

The functional equation (64) can be solved numerically with the similar technique I used before. Once we have the prices of the consumption claim, we can proceed to price other assets. The pricing formulas for the dividend claim and the real risk-free bond are respectively

$$\frac{SP_t}{D_t}(p_t) = E_t[\beta^{\frac{1-\rho}{1-\theta}}(\frac{C_{t+1}}{C_t})^{-\rho}[\frac{\frac{SP_{t+1}}{C_{t+1}}(p_{t+1})+1}{\frac{SP_t}{C_t}(p_t)}]^{\frac{1-\rho}{1-\theta}-1}(\frac{SP_{t+1}}{D_{t+1}}(p_{t+1})+1)] \tag{65}$$

and

$$\frac{1}{1+r(p_t)} = E_t[\beta^{\frac{1-\rho}{1-\theta}}(\frac{C_{t+1}}{C_t})^{-\rho}[\frac{\frac{SP_{t+1}}{C_{t+1}}(p_{t+1})+1}{\frac{SP_t}{C_t}(p_t)}]^{\frac{1-\rho}{1-\theta}-1}] \tag{66}$$

Foreign asset prices can be computed following exactly the same procedure.

As shown in Backus, Foresi and Telmer (2001), among many others, under complete markets, the change in the real exchange rate between Home and Foreign is simply the ratio of two stochastic discount factors

$$\frac{Q_{t+1}}{Q_t} = \frac{m^*_{t+1}}{m_{t+1}} \tag{67}$$

With equations (64)–(67) and their Foreign counterparts plus the shock process specified in section 3.3, I can then numerically evaluate the model.

Table 8 contains parameter values being used in the simulation. Several points should be made here. 1. Note that $\theta = 0.2$, which implies that $IES = 5$. This seems to be at odds with empirical studies on IES using Macro data, most notably Hall (1988). But as pointed out by Bansal and Yaron (2004), Hall's regression may have a severe downward bias. Indeed, running a Hall-type regression within this model would reach the conclusion that $IES = 0$ although the actual IES is assumed to be 5. Studies based on micro-level data have little consensus on the magnitude of IES, either. In

Gourinchas and Parker (2002), the point estimate of IES is 2, but $IES = 5$ is well within 2 standard deviations around the mean. 2. The coefficient of relative risk aversion ρ is set to be 4 instead of 5. This adjustment is made simply to match the equity premium correctly. 3. The process of p is assumed to be more persistent than before. This is to increase the volatility of stock prices.

[INSERT TABLE 8 HERE]

Table 9 reports price/dividend ratios under different p. As expected, an increase in p now causes a drop in stock prices. But also note that stock prices now respond less strongly to a change in the probability of disasters p and this eventually causes stock prices to be less volatile in the model compared to the actual data.

[INSERT TABLE 9 HERE]

Table 10 reports summary statistics from simulation of 1,000,000 years of data conditional on no disasters. As we can see, the risk-free rate is still low and the equity premium is at around 6% per year. The forward discount puzzle is also there. The economics behind these results is the same as before. Note that since we did not explicitly model the nominal aspect of the economy, here the forward discount puzzle is completely in terms of real variables. Stock prices become a little bit too smooth and exchange rates become too volatile. But if we take a closer look at the determinants of the volatility of exchange rates, things may not be as bad as they look. Taking logarithms on both sides of equation (67) yields

$$q_{t+1} - q_t = \rho(\log \frac{C_{t+1}}{C_t} - \log \frac{C^*_{t+1}}{C^*_t}) + (\frac{\rho - \theta}{1 - \theta})(\log(\frac{\frac{SP_{t+1}}{C_{t+1}} + 1}{\frac{SP_t}{C_t}}) - \log(\frac{\frac{SP^*_{t+1}}{C^*_{t+1}} + 1}{\frac{SP^*_t}{C^*_t}})) \tag{68}$$

The first term on the right hand side of equation (68) is the familiar consumption growth differential. Given that we are assuming that there is no correlation between Home consumption growth and Foreign consumption growth, this term alone generates more volatility in exchange rates than before. The second term reflects the effects of future consumption growth on today's exchange rate, which is a new feature due to the recursive nature of the Epstein-Zin-Weil preferences. The assumption that there is no correlation of consumption growth between Home and Foreign forever causes exchange rates to be much more volatile than it would otherwise be. Although a thorough analysis is beyond the scope of this paper, it is entirely possible to substantially reduce the volatility of exchange rates by imposing some long-term correlation in consumption growth between Home and Foreign. In fact, this is exactly what Colacito and Groce (2005) do in their paper. With a mean-reverting probability of disasters, all results in table 3 and 4 are still true. To save space, the results are not reported here.

[INSERT TABLE 10 HERE]

In addition to the results established above, equation (68) also implies a less-well-known empirical regularity documented in Hau and Rey (2006)–higher returns in the Home equity market relative to the Foreign equity market are associated with a depreciation of the Home currency (see table 2, 3 and 4 in Hau and Rey (2006)) . When both the coefficient of risk aversion and the IES are greater than 1, $\frac{\rho-\theta}{1-\theta}$ is positive in equation (68). Other things being equal, relatively higher Home equity market returns, i.e. $\log(\frac{\frac{SP_{t+1}}{C_{t+1}}+1}{\frac{SP_t}{C_t}}) > \log(\frac{\frac{SP^*_{t+1}}{C^*_{t+1}}+1}{\frac{SP^*_t}{C^*_t}})$, necessarily cause a Home depreciation. This can be considered as a stock market version of the UIP condition discussed in Hau and Rey (2006).

Unlike Hau and Rey (2006), which provides a micro-structure explanation for the stock market version UIP condition, the economics in this model is more in line with the so-called "asset price" view of the exchange rate. Equation (68) clearly shows that real exchange rate depends not only on the current consumption differentials, but also on all future consumption differentials that are represented by the prices of consumption claims. In the time separable utility case, $\frac{\rho-\theta}{1-\theta}$ is equal to 0, therefore the asset price aspect of the real exchange rate is completely shut off. When $\frac{\rho-\theta}{1-\theta} > 0$, which is the case under the parameter values I specified in order to match other asset prices, current consumption and future consumption are substitutes. A drop in expected future Home consumption increases the marginal utility of consumption today which in turn causes a Home real appreciation now. A drop in expected future Home consumption also lowers the price of consumption claim as discussed earlier. But if the probability of such a drop is mean-reverting, the Home exchange rate will on average depreciate next period while stock prices will on average rise. In terms of returns, higher returns in the Home equity market are associated with a Home depreciation. Therefore, by having both the Epstein-Zin-Weil preferences and rare disasters, this model can potentially explain why the UIP condition fails to hold in the bond market but still survives in the stock market.

Of course, the above model is still far from satisfactory, but at least demonstrates that most intuitions we have developed for the standard time-separable CRRA utility function is robust when the coefficient of risk-aversion and the IES are separately parameterized.

8 Concluding Remarks

Introducing rare disasters into an otherwise standard RBC model makes a lot of progress in terms of explaining several long-standing puzzles in macroeconomics, finance and international finance. Within a single model, this paper can explain the equity premium puzzle, the risk-free rate puzzle, excess volatility, the forward discount puzzle and the volatility mismatch puzzle both qualitatively and quantitatively. If the underlying economics of this paper captures some of the truth, it then confirms the conjecture that many economists have had for a long time that many macroeconomics, finance and international finance puzzles may share a common cause.[32] Another strength of this paper is that almost all standard assumptions in macroeconomics are maintained and many of its results are easily solvable by hand. This makes the economics of this paper more transparent and future extensions easier.

Looking forward, the rare-disaster approach has several advantages which I consider appealing. First of all, rare disasters certainly can be taken literally. There have been plenty in the past and we are not disaster-free yet. Massive terrorist attack, catastrophic climate change, financial turmoil, for example, are not completely impossible in the foreseeable future. Second, methodologically speaking, the rare-disaster approach is more about how we should think about risks. Random shocks following the Gaussian distribution may be a natural place to start with but may not be the correct assumption. Indeed, quantitative implications of a standard CRRA utility function are very sensitive to distributional assumptions.[33] Rietz (1988), Barro(2006), Gabaix (2007) and this paper all point to the fact that a minor change in the assumption about the distribution of shocks could lead a long way toward solving those quantitative puzzles. Weitzman (2007) goes even a step further: the tail-fattening effect in that paper not only eliminates the equity premium puzzle, but also generates a counter-puzzle: equity premium can go to infinity with the standard CRRA utility. The recent surge of interests in understanding of the tail properties of random shocks reveals that a lot more needs to be learned in the future about how subjective evaluation of risks is formed and how it evolves. Third, the rare-disaster approach may have the potential of matching both low-frequency data and high-frequency

[32] See Obstfeld and Rogoff(1996) for some discussion on this point.

[33] See Geweke(2001) for several nice examples and Weitzman(2007) for a more thorough analysis.

data at the same time. People's preferences may not be changing at very high frequency, nor is the marginal investor whose pricing kernel eventually prices all assets, so any theories that rely on different assumptions about preferences and market structures may have a hard time to match high-frequency data. Noise traders and information problem may be able to affect short-term performance of assets but it is less clear what the long-term impacts these elements could have on risks and returns, so they may not be able to match low-frequency movements. But if the underlying risks or perceived risks are changing overtime, it is not inconceivable that this could be the driving force behind both high-frequency and low-frequency movements in asset prices. Finally, the rare-disaster approach can be a technically convenient way to model shocks with fatter tails and conditional heteroscadesticity, which are the key components to generate sizable time-varying risk premium under moderate risk-aversion.

A potential critique of the rare-disaster approach could be that we don't know how to measure the probability of rare disasters and thereby making this approach untestable or unverifiable. Certainly, more needs to be done before we can have a better idea of what subjective risks really look like. However, if this is a critique to the rare-disaster approach, it may well be a critique applied to a wide range of models. The way we judge the success of RBC models is not whether we can identify real shocks so that we can map those shocks to economic ups and downs, but rather simply looking at if the artificial data generated can match important moments in the actual data. We certainly have not been able to measure stand-in traders' "surplus consumption ratio", but we still consider habit-formation a useful way to think about asset prices. Similarly, we may not be able to measure disaster probabilities directly for now, but as long as the model can match data reasonably well, it may not a be a bad model after all.

A natural next step is to recover probabilities of disasters from some asset prices such as options and then to see whether other asset prices move in the right direction as this model predicts. A more careful study of the asset pricing implications under the Epstein-Zin-Weil preferences is needed and seems promising. A richer structure of the goods market is also worthy of pursuit since the terms of trade effect is completely missing in the current model.

9 Appendix

A Model Derivation

A Home representative agent's maximization problem is the following

$$
\begin{aligned}
& U_t = E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{C_s^{1-\rho}}{1-\rho}+\frac{\chi}{1-\epsilon}\left(\frac{M_s}{P_s}\right)^{1-\epsilon}\right]\right\} \\
s.t. \quad & P_tC_t + M_t + \sum_{s_{t+1}} Q(s_{t+1}|s_t)B(s_{t+1}) \leq P_tY_t + M_{t-1} + B(s_t) + T_t \\
& c_{it} \leq y_{it} \quad \forall i \in [0,\alpha]
\end{aligned}
$$

where $C = \exp(\int_0^1 \log c_i di)$, $Y_t = \exp(\int_0^1 \log y_{it} di)$ and $P = \exp(\int_0^1 \log p_i di)$.

The FOCs are

$$Q(s_{t+1}|s_t) = \beta\pi(s_{t+1}|s_t)\frac{U_{c_i}(s_{t+1})}{U_{c_i}(s_t)}\frac{p_{i,t}}{p_{i,t+1}} \tag{A.1}$$

$$1 - \chi C_t^{\rho}[\frac{M_t}{P_t}]^{-\epsilon} = E_t m_{t+1}\frac{P_t}{P_{t+1}} \tag{A.2}$$

$$c_{it} = y_{it} \quad \forall i \in [0,\alpha] \tag{A.3}$$

where $\pi(s_{t+1}|s_t)$ is the probability of state s_{t+1} conditional on s_t.Equation (A.1) is the Euler equation for good i, equation (A.2) is the optimal condition for money demand and equation (A.3) simply says that non-tradable consumption is equal to its endowment.

A Foreign representative agent solves the following optimization problem

$$
\begin{aligned}
& U_t^* = E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{C_s^{*1-\rho}}{1-\rho}+\frac{\chi}{1-\epsilon}\left(\frac{M_s^*}{P_s^*}\right)^{1-\epsilon}\right]\right\} \\
s.t. \quad & P_t^*C_t^* + M_t^* + \sum_{s_{t+1}} Q(s_{t+1}|s_t)B^*(s_{t+1})/\mathcal{E}_t \\
\leq \quad & P_t^*Y_t^* + M_{t-1}^* + B^*(s_t)/\mathcal{E}_t + T_t^* \\
& c_{it}^* \leq y_{it}^* \quad \forall i \in [0,\alpha]
\end{aligned}
$$

The FOCs are

$$Q(s_{t+1}|s_t) = \beta\pi(s_{t+1}|s_t)\frac{U_{c_i^*}(s_{t+1})}{U_{c_i^*}(s_t)}\frac{p_{i,t}^*}{p_{i,t+1}^*}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{A.4}$$

$$1 - \chi C_t^{*\rho}[\frac{M_t^*}{P_t^*}]^{-\epsilon} = E_t m_{t+1}^* \frac{P_t^*}{P_{t+1}^*} \tag{A.5}$$

$$c_{it}^* = y_{it}^* \quad \forall i \in [0, \alpha] \tag{A.6}$$

From equations (A.1) and (A.4). we can derive the risk-sharing condition

$$\frac{U_{c_i}(s_{t+1})}{U_{c_i}(s_t)}\frac{p_{i,t}}{p_{i,t+1}} = \frac{U_{c_i^*}(s_{t+1})}{U_{c_i^*}(s_t)}\frac{p_{i,t}^*}{p_{i,t+1}^*}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{A.7}$$

Iterating equation (A.7) back to period 0 yields

$$\frac{U_{c_i}(s_{t+1})}{U_{c_i}(s_0)}\frac{p_{i,0}}{p_{i,t+1}} = \frac{U_{c_i^*}(s_{t+1})}{U_{c_i^*}(s_0)}\frac{p_{i,0}^*}{p_{i,t+1}^*}\frac{\mathcal{E}_0}{\mathcal{E}_{t+1}} \tag{A.8}$$

Since LOOP always holds for tradable goods, for tradable goods equation (A.8) can be further simplified to

$$\frac{U_{c_i}(s_{t+1})}{U_{c_i}(s_0)} = \frac{U_{c_i^*}(s_{t+1})}{U_{c_i^*}(s_0)} \quad \forall i \in (0, 1] \tag{A.9}$$

Equation (A.9) is the familiar perfect risk-sharing condition, but it only holds for tradable goods. If $U_{c_i}(s_0) = U_{c_i^*}(s_0)$, marginal utility of tradable goods consumption is always equalized across countries. More generally, marginal utility of tradable goods consumption is always proportional across countries. To save notations, I am going to assume $U_{c_i}(s_{t+1}) = U_{c_i^*}(s_{t+1})$ for all tradable goods. It is straightforward to extend this model to more general case, the results won't change.

Given $U_{c_i}(s_{t+1}) = U_{c_i^*}(s_{t+1})$ for all tradable goods and taking into account that $c_{it} = y_{it} \equiv A_t$ and $c_{it}^* = y_{it}^* \equiv A_t^*$ for all non-tradable goods, also noting that in a symmetric equilibrium tradable goods consumption will be exactly the same for all varieties, we can rewrite equation (A.9) as

$$c_{N,t}^{\alpha(1-\rho)} c_{T,t}^{-\alpha(1-\rho)-\rho} = (c_{N,t}^*)^{\alpha(1-\rho)} (c_{T,t}^*)^{-\alpha(1-\rho)-\rho} \tag{A.10}$$

where $c_{i,t} = c_{N,t}$ for $\forall i \in [0, \alpha]$ and $c_{i,t} = c_{T,t}$ for $\forall i \in (\alpha, 1]$ and similar definition for Foreign.

Equation (A.10) is the perfect risk-sharing condition for any country pairs in a symmetric equilibrium. Goods market clearing condition for tradable goods requires that for any tradable good i the following must be true

$$\int\limits_{w \in \Omega} c_{i\omega t} d\omega = \int\limits_{w \in \Omega} y_{i\omega t} d\omega \equiv \int\limits_{w \in \Omega} A_{\omega t} d\omega \quad \forall i \in (0, 1] \tag{A.11}$$

Substituting equation (A.10) into equation (A.11) gives us

$$c_{it} = \frac{A_t^{-\gamma}}{\int\limits_{w \in \Omega} A_{\omega t}{}^{-\gamma} d\omega} \int\limits_{w \in \Omega} A_{\omega t} d\omega \quad \forall i \in (0, 1] \tag{A.12}$$

where $\gamma = \frac{(1-\rho)\alpha}{(\rho-1)\alpha-\rho} > 0$ when $\rho > 1$. Foreign has similar equations. It is not hard to derive from the first order conditions that relative price of good i and good j must satisfy

$$\frac{p_i}{p_j} = \frac{c_j}{c_i} \tag{A.13}$$

Under symmetry, we have $p_i = p_N$ for $\forall i \in [0, \alpha]$ and $p_i = p_T$ for $\forall i \in (\alpha, 1]$ and therefore $P = p_N^{\alpha} p_T^{1-\alpha}$. Similar equations hold for Foreign.

B The Fiscalist Version

This version is adapted from Cochrane (2005). Here is only a sketch of the model. Please refer to Cochrane (2005) for more details.

Everything is the same as in the monetarist version except the money-in-utility assumption is now replaced by cash-in-advance constraint. So everything else is exactly the same except the money demand function is replaced by the following two equations:

$$M_t = P_t C_t \tag{A.14}$$

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} E_t(m_{t,t+j} \widetilde{s}_{t+j}) \tag{A.15}$$

where B_{t-1} is outstanding nominal government bond, $\widetilde{s}_{t+j}$ is the government real primary surplus in period $t+j$ and $m_{t,t+j}$ is the stochastic discount factor between period t and $t+j$. They are equation 3 and 4 in the Cochrane(2005). Equation (A.14) is the familiar cash-in-advance constraint and equation (A.15) says that the real value of outstanding nominal government debt should be equal to the discounted future government real primary surplus.

Further assume that asset market will reopen after purchase for consumption goods has been made, this makes demand for cash drop to 0 and we are left with only equation (A.15). This is a cashless economy, however price level is still well-defined. Now the price level depends crucially on the current and future primary surplus for given outstanding government bond. Or put it differently, price level is determined by fiscal policy now. Equation (A.15) can also be written into a recursive form

$$\frac{B_{t-1}}{P_t \widetilde{s}_t}(p_t) = 1 + E_t[m_{t+1} \frac{\widetilde{s_{t+1}}}{\widetilde{s}_t} \frac{B_t}{P_{t+1}\widetilde{s_{t+1}}}(p_{t+1})] \tag{A.16}$$

It is easy to see equation (A.16) is very similar to equation (48) in the monetarist version. That's why the fiscalist version can generate identical inflation process as in the monetarist version if fiscal policy is chosen in a such way that $\frac{\widetilde{s_{t+1}}}{\widetilde{s}_t} \equiv (\frac{C_{t+1}}{C_t})^{\frac{\varrho}{\epsilon}} \frac{M_t}{M_{t+1}}$ at any period t and in any state s_t.

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Table 1: Parameter Values

Parameter	Value	Parameter	Value
β	.97	α	0.95
ρ	5	η	0.95
g	1.89%	σ_ϵ	0.0025
σ_u	1.5%	p_{bench}	0.017
b	0.42	q	0.4
F	$\frac{1}{1-b}$	σ_ω	11.2%
λ	0.5	δ	0.2
Implied Value			
γ	3.16	ϕ	0.80

This table contains parameter values used in the ben--chmark calibration. Two composite parameters are defined as: $\gamma = \frac{(1-\rho)\alpha}{(\rho-1)\alpha-\rho}$ and $\phi = \frac{\alpha}{\rho-(\rho-1)\alpha}$.

Table 2: Simulation Results

	Model				Data	
Asset Prices	Cons Claim	Div Claim	NT Claim	Levered	**Post War**	**Long**
True r^f	0.7	0.7	0.7	0.7	**N/A**	**N/A**
$\sigma(truer^f)$	4.0	4.0	4.0	4.0	**N/A**	**N/A**
r^f	2.3	2.3	2.3	2.3	**.094**	**2.92**
$\sigma(r^f)$	2.2	2.2	2.2	2.2	**2.1**	**4.7**
$r^e - r^f$	3.4	4.4	4.3	11.4	**6.69**	**3.90**
$\sigma(r^e - r^f)$	7.0	13.3	5.6	25.6	**15.7**	**18.0**
$\frac{r^e-r^f}{\sigma(r^e-r^f)}$	.49	.33	.77	.44	**.43**	**.22**
$\exp(p-d)$	31.1	27.3	23.4	N/A	**24.7**	**21.1**
$\sigma(p-d)$	.23	.20	.19	N/A	**.26**	**.27**
	Model	Data (Post Bretton Woods 1973-2007)				
Exchange Rates		**US**	**UK**	**Japan**	**Euro Area**	
$\sigma(\Delta q)$	8.4	**6.3(4.8)**	**6.3(5.2)**	**11(9.6)**	**6.8(5.9)**	
$\sigma(\Delta e)$	8.9	**7.3(5.7)**	**6.5(4.7)**	**11(9.6)**	**6.8(6.0)**	
The Fama Regression						
β_1	-1.88					
$\sigma(\beta_1)$	1.00					

This table reports the summary statistics from the simulated data under the benchmark calibration. Numbers in bold are historical statistics from Campbell and Cochrane (1999), Barro (2006) and the author's calculation. See footnote 24 for more details. The exchange rates are trade-weighted exchange rates published by BIS. The standard deviations of both HP-filtered (in parentheses) and non-HP-filtered data are reported. All exchange rates are from June 1st of each year.

Table 3: Autocorrelations

	Model				Data	
Variable	Cons Claim	Div Claim	NT Claim	Levered	**Post War**	**Long**
$r^e - r^f$						
$AC(1)$	-.07	-.004	-.06	-.04	**-.11**	**.05**
$AC(2)$	-.02	.05	-.02	-0.002	**-.28**	**-.21**
$p - d$						
$AC(1)$	.93	.93	.93	N/A	**.87**	**.78**
$AC(2)$	.88	.88	.88	N/A	**.77**	**.57**

This table reports autocorrelations of excess returns to stocks and (log) price/dividend ratios. Numbers in bold are historical statistics from Campbell and Cochrane(1999).

Table 4: Cross-Correlations

	Model			Data	
Variable	Cons Claim	Div Claim	NT Claim	**Post War**	**Long**
$p_t - d_t, r^e_{t+j}$					
$j = 1$	-.28	-.14	-.29	**-.42**	**-.20**
$j = 2$	-.25	-.13	-.26	**-.25**	**-.21**
$r^e_t, \|r^e_{t+j}\|$					
$j = 1$	-.05	.02	-.05	**-.32**	**-.15**
$j = 2$	-.003	.04	-.006	**-.14**	**.03**
$p_t - d_t, \|r^e_{t+j}\|$					
$j = 1$	-.25	-.08	-.27	**-.16**	**-.12**
$j = 2$	-.22	-.07	-.24	**.09**	**.02**

This table reports cross-correlations among (log) price/dividend ratios, returns to stocks and absolute value of returns to stocks. Numbers in bold are historical statistics from Campbell and Cochrane(1999).

Table 5: Price/Dividend Ratios

p	Cons Claim	Div Claim	NT Claim
0.001	18.2	17.4	15.3
0.005	20.7	19.4	16.9
0.009	24.2	22.1	19.2
0.013	28.6	25.4	22.0
0.017	33.4	29.1	25.0
0.021	38.2	32.7	28.0
0.025	42.1	35.6	30.3

This table reports price/dividend ratios for different disaster probabilities under the benchmark calibration.

Table 6: Alternative Specifications

Variable	Benchmark	Actual Disasters	Naive Monetary Policy
r^f	2.3	1.8	1.8
$\sigma(r^f)$	2.2	5.9	11.9
Consumption Claim			
$r^e - r^f$	3.4	3.1	3.9
$\sigma(r^e - r^f)$	7.0	8.5	3.6
$\exp(p - d)$	31.1	31.1	31.1
$\sigma(p - d)$	.23	.23	.23
Dividend Claim			
$r^e - r^f$	4.4	4.0	4.9
$\sigma(r^e - r^f)$	13.3	14.1	12.8
$\exp(p - d)$	27.3	27.3	27.3
$\sigma(p - d)$	.20	.20	.20
NT Claim			
$r^e - r^f$	4.3	4.0	4.8
$\sigma(r^e - r^f)$	5.6	7.4	4.9
$\exp(p - d)$	23.4	23.4	23.3
$\sigma(p - d)$	.19	.19	.19
Levered			
$r^e - r^f$	11.4	12.7	9.0
$\sigma(r^e - r^f)$	25.6	31.1	9.2
Exchange Rates			
$\sigma(\Delta q)$	8.4	29	8.4
$\sigma(\Delta e)$	8.9	27	18.4
The Fama Regression			
β_1	-1.88	-1.10	-1.90
$\sigma(\beta_1)$	1.00	2.93	.344

This table reports the summary statistics from the simulated data when actual disasters are allowed and the naive monetary policy is adopted.

Table 7: Different Parameter Values

Variable	Benchmark	Low ρ	Low q	High g	Low η	low α	High β
		$\rho = 4$	$q = 0$	$g = 0.025$	$\eta = 0.9$	$\alpha = 0.8$	$\beta = 0.98$
r^f	2.3	3.9	-.001	5.5	2.2	9.8	1.3
$\sigma(r^f)$	2.2	0.9	2.4	1.7	1.6	0.3	2.4
Consumption Claim							
$r^e - r^f$	3.4	2.6	5.8	3.5	3.1	.8	3.3
$\sigma(r^e - r^f)$	7.0	4.8	11.5	6.3	5.7	3.0	7.5
$\exp(p - d)$	31.1	22.8	31.1	16.9	31.8	12.0	47.8
$\sigma(p - d)$	.23	.14	.23	.18	.15	.06	.23
Dividend Claim							
$r^e - r^f$	4.4	3.2	6.8	4.5	4.1	1.9	4.3
$\sigma(r^e - r^f)$	13.3	12.7	15.8	13.4	12.8	12.7	13.3
$\exp(p - d)$	27.3	23.0	27.3	15.7	28.1	11.3	39.5
$\sigma(p - d)$	.20	.12	.20	.15	.13	.03	.20
NT Claim							
$r^e - r^f$	4.3	3.1	6.7	4.4	4.0	1.9	4.2
$\sigma(r^e - r^f)$	5.6	4.2	10.0	5.2	4.7	2.4	5.9
$\exp(p - d)$	23.4	20.2	23.4	14.4	24.2	10.5	32.2
$\sigma(p - d)$	.19	.12	.19	.15	.12	.03	.19
Levered							
$r^e - r^f$	11.4	6.2	19.4	9.8	7.3	1.7	15.1
$\sigma(r^e - r^f)$	25.6	11.4	42.1	18.0	13.5	6.3	48.1
Exchange Rates							
$\sigma(\Delta q)$	8.4	7.0	8.4	8.4	8.4	4.7	8.4
$\sigma(\Delta e)$	8.9	7.1	9.0	8.7	8.7	4.7	9.1
The Fama Regression							
β_1	-1.88	-1.51	-2.93	-1.11	-3.20	-.75	-2.53
$\sigma(\beta_1)$	1.00	1.45	1.60	0.69	1.53	1.31	1.15

This table reports the summary statistics from the simulated data for different parameter values.

Table 8: Parameter Values under the Epstein-Zin-Weil Preferences

Parameter	Value	Parameter	Value
β	.97	α	1
ρ	4	η	0.98
θ	0.2	σ_ϵ	0.0025
g	1.89%	p_{bench}	0.017
σ_u	1.5%	σ_ω	11.2%
b	0.42	δ	0.2

This table contains parameter values for simulation under the Epstein-Zin-Weil preferences.

Table 9: Price/Dividend Ratios under the Epstein-Zin-Weil Preferences

p	Cons Claim	Div Claim
0.001	34.6	37.9
0.005	33.3	36.5
0.009	31.5	34.6
0.013	29.7	32.6
0.017	28.0	30.6
0.021	26.4	28.9
0.025	25.1	27.4
0.029	24.1	26.3
0.033	23.4	25.6

This table reports price/dividend ratios for different disaster probabilities under the Epstein-Zin-Weil preferences.

Table 10: Simulation Results under the Epstein-Zin-Weil Preferences

Variable	Cons Claim	Div Claim
Asset Prices		
r^f	0.3	0.3
$\sigma(r^f)$	1.9	1.9
$r^e - r^f$	5.6	6.2
$\sigma(r^e - r^f)$	3.7	12.6
$\frac{r^e - r^f}{\sigma(r^e - r^f)}$	1.51	.49
$\exp(p - d)$	31.4	28.7
$\sigma(p - d)$	.13	.13
Exchange Rates		
$\sigma(\Delta q)$	22.7	
The Fama Regression		
β_1	-2.20	
$\sigma(\beta_1)$	0.28	

This table reports the summary statistics from the simulated data under the Epstein-Zin-Weil preferences discussed in section 7. 1,000,000 years of data are simulated conditional on no disasters.

Graph 1: Impulse-Response to a Sudden Drop in p